Problem 1 (3 points)
Consider the following problem.

$$\text{Max } z = 3x_1 + x_2$$

$$\text{s.t.} \quad x_1 - x_2 \geq 2$$
$$2x_1 - x_2 \geq 6$$

$$x_1 \geq 0 \qquad x_2 \geq 0$$

1. Use TORA to graph the feasible region of this problem. Use this graph to identify the **extreme points** of the feasible region and any **direction of unboundedness** if applicable.

2. Represent the point $x_1=5$, $x_2 = 1$ as a convex combination of extreme points and a direction of unboundedness (if any).

Problem 2 **(4 points)**
Solve the following problem using the revised simplex method. Perform at most two iterations.

$$\text{Max } z = 2x_1 - x_2 + x_3$$

$$\begin{aligned} \text{s.t.} \quad & x_1 - x_2 + 3x_3 \leq 4 \\ & 2x_1 + x_2 \leq 10 \\ & x_1 - x_2 - x_3 \leq 7 \\ & x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \end{aligned}$$

Problem 3 (4 points)

Use the upper bound simplex method to solve the following problem starting with $x_1 = x_3 = 0$ and $x_2 = 1$.

$$\text{Max } z = 3x_2 - 2x_3$$

$$\begin{aligned}
\text{s.t.} \quad & x_2 - 2x_3 \le 1 \\
& 2x_1 + x_2 + 2x_3 \le 8 \\
& x_1 \le 1 \\
& x_2 \le 3 \\
& x_3 \le 2
\end{aligned}$$

$$x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0$$

Problem 4 (3 points)
Use the dual simplex method to solve the following problem.

$$\text{Min } z = 3x_1 + 2x_2 + 4x_3$$

s.t.

$$3x_1 + x_2 + 2x_3 \geq 4$$

$$6x_1 + 3x_2 + 5x_3 \geq 10$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$$

Problem 5 (12 points)

Consider the following problem.

$$\text{Max } z = 3x_1 + 2x_2$$

$$\text{s.t.} \quad \begin{aligned} 3x_1 + x_2 &\leq 12 \\ x_1 + x_2 &\leq 6 \\ 5x_1 + 3x_2 &\leq 27 \end{aligned}$$

$$x_1 \geq 0 \qquad x_2 \geq 0$$

1. Use TORA to graph this problem. Deduce the optimal solution of this problem indicating the basic and non basic variables as well as the value of the objective function.

2. Deduce the optimal simplex tableau.

3. Write the dual problem.

4. Deduce the optimal solution of the dual.

5. Answer the following question independently of each others. You don't need to give the new solution (if any).

- What happens to the optimal solution if the right hand side of the first constraint is increased by 4?

- What happens to the optimal solution if the objective function coefficient of x_2 is increased by 3?

- What happens to the optimal solution if a new decision variable x_3 is added to the problem where $c_3=3$ and $a_3^T = (-1\ -2\ 3)$?

- What happens to the optimal solution if the constraint

$$x_1 + 2x_2 \leq 18$$

  is added to the problem?

Problem 6 (4 points)
Solve the following problem using the two-phase method.

$$\text{Max} \quad z = 8\,x_1$$

s.t.

$$x_1 + x_2 \geq 4$$

$$2\,x_1 + x_2 \leq 1$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Problem 7 (5 points)
Consider the following LP model.

$$\text{Max } z = 8x_1 + 24x_2$$

s.t.

$$x_1 + 2x_2 \leq 10$$
$$2x_1 + x_2 \leq 10$$

$$x_1 \geq 0, \; x_2 \geq 0$$

Let s_1 and s_2 denote the slack variables of the first and second constraints, respectively.

The optimal solution of this problem is $x_1 = 5$, $x_2 = 0$, $s_1 = 0$, $s_2 = 5$, and $z = 120$. The corresponding optimal simplex tableau follows.

	-4	0	-12	0	-120
	x_1	x_2	s_1	s_2	
x_2	0.5	1	0.5	0	5
s_2	1.5	0	-0.5	1	5

Apply parametric programming to the linear program with $\Delta c^T = (1 \quad -2)$; that is, with

$$z(\theta) = 8x_1 + 24x_2 + \theta(x_1 - 2x_2)$$

$$= (8+\theta)x_1 + (24 - 2\theta)x_2,$$

and

$$0 \leq \theta \leq 10.$$

Problem 8 (5 points)
Consider the following LP model.

$$\text{Max} \quad z = 3x_1 + x_2$$

s.t.

$$x_1 + x_2 \leq 4$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Starting from the initial solution $x_1=1$, $x_2 = 2$, perform two iterations of the interior point algorithm. Explain the role of each step of the algorithm.