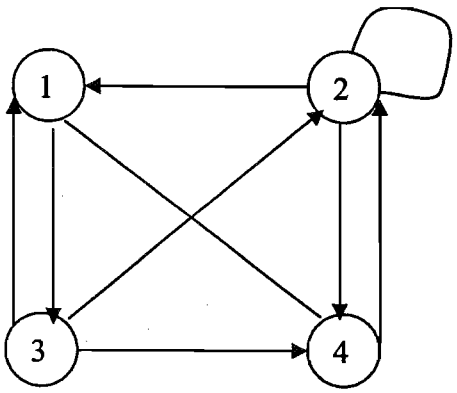
1. **(11 pts)** Consider the following graph.



a) **(1.5 pts)** Write the adjacency matrix of the graph.

b) **(1.5 pts)** Write the incidence matrix.

c) **(1.5 pts)** Write the adjacency list.

d) **(1 pt)** Which of the methods of parts (a)-(c) is the most efficient one for representing the above graph? Why?

e) **(1.5 pts)** Does the graph have a geometric dual? Why? If it does, draw the dual.

f) **(1 pt)** Does the graph have an Euler cycle? Why?

g) **(1 pts)** Find the in-degree and out-degree of each node.

h) **(2 pts)** Is the graph:
i) directed, undirected, or mixed? Why?

ii) acyclic? Why?

iii) a bi-partite? Why?

iv) a forest? why?

2. **(3 pts)** Answer the following questions.
a) Draw a graph with 4 nodes whose degrees are 2, 3, 3, and 6.

b) Show that there is an even number of odd-degree nodes in a graph.

c) Show that in a directed graph the sum of in-degrees and the sum of out-degrees are equal.

3. **(6 pts)** Suppose that the cost of a new care is 6,000 KD. The annual operating cost and the resale value of a used car are given below. Assume that a student buys a new car now and keeps it for the next four years.

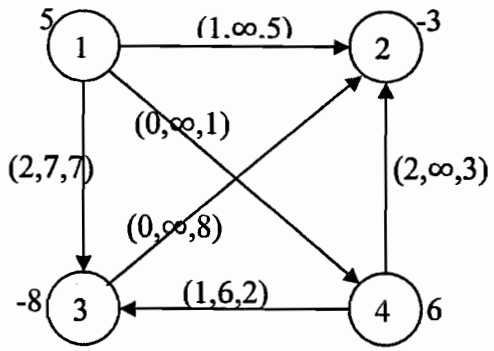
Age of Car	Resale Value	Operating Cost
1	4,500 KD	100 KD (year 1)
2	3,500 KD	200 KD (year 2)
3	2,200 KD	300 KD (year 3)
4	1,500 KD	400 KD (year 4)

a) **(2 pts)** Construct a network that can be used to find a car replacement policy with the minimum cost during the four years.

b) **(2 pts)** Use the network of part (a) to present a linear program whose solution gives a car replacement policy with the minimum cost.

c) **(2 pts)** Apply the Dijkstra's algorithm to the network of part (a) to determine the best car replacement policy for the student.

4. **(8 pts)** Consider the following min-cost flow problem. The numbers beside each arc show the lower and upper bound capacities on flow and the unit cost. The number beside each node gives the supply/demand.



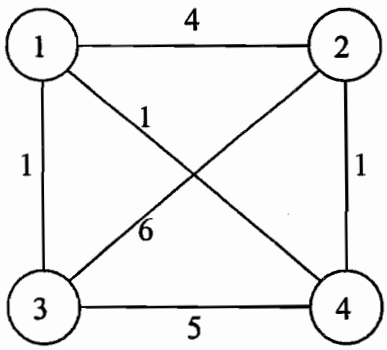
a) **(3 pts)** Remove the lower and upper bound capacities on flows and give the resulting network.

b) **(1 pt)** Give an initial feasible flow for the resulting network of part (a).

c) **(3 pt)** Use the initial feasible flow of part (b) as a starting solution and the negative cycle method to find the optimal flow for the resulting network of part (a).

d) **(1 pt)** Using the optimal flow of part (c), find the optimal flow for the original network.

5. **(6 pts)** Consider the following network where the number beside each arc shows the arc distance.



a) **(1.5 pts)** Use the Prim algorithm to find the maximum spanning tree.

b) **(1.5 pts)** Find the Steiner tree among nodes 2, 3, and 4.

c) **(3 pts)** If a post office is located at node 3, what is the solution to the postman problem?

6. **(6 pts)** Suppose that there are three different computers and four different disk drives. The compatibility of each computer with the disk drives are indicated by (x) in the following table.

Computer	A	B	C	D
1	x	x		x
2		x	x	
3	x		x	x

a) **(1 pt)** Show this compatibility problem by a matching bi-partite.

b) **(1.5 pts)** Construct the matching network for the problem.

c) **(2 pts)** Use the network of part (b) to find the maximum number of computers with compatible disk drives.

d) **(1.5 pts)** Use Hall's marriage theorem to see if for each computer there is a compatible disk drive.