1. Claims arrive at an insurance company according to a Poisson process with rate λ per year. The claim sizes are independent random variables and have the common discrete distribution

$$P(X=k)= a_k = \frac{-\alpha^k}{k\ \ln(1-\alpha)} \qquad \text{for } k=1,\ 2,\ \ldots,$$

where α is a constant between 0 and 1. Find the mean and the variance of the total amount claimed during a given year.

2. Consider the following transition probability matrix:

$$P = \begin{vmatrix} 0.3 & 0.5 & 0 & 0 & 0 & 0.2 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0.7 \\ 0.1 & 0 & 0.1 & 0 & 0.8 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{vmatrix}$$

(a) Which states are transient?

(b) Which states are recurrent?

(c) Identify all closed sets of states.

2. Continued

(d) Calculate the expected number of visits to state 3 given that $X_0=1$.

(e) Compute the limiting probabilities π_j for j= 0, 1, ,2 , 3, 4, 5.

3. Three white and five red balls are distributed in two urns in such a way that each contains four balls. We say that the system is in state i, i=0, 1, 2, 3, if the first urn contains i white balls. Each period, we draw one ball from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn. Let X_n denote the state of the system after the nth period.

(a) Explain why $\{X_n, n=1, 2, 3, \ldots\}$ is a Markov chain.

(b) Calculate the one-step transition probability matrix.

3.

4. Arrivals to a repair shop occur according to a Poisson process with rate $\lambda = 8$ per hour.

(a) What is the probability that there will be 4 or less arrivals during the first 30 minutes?

(b) Estimate the probability that there will be 70 or more arrivals during an 8-hour day?

(c) Starting at 8:00 A.M., what is the expected time at which the fifth repair occurs?

4. Continued

(d) What is the probability that two or more events occur between 1 P.M. and 2 P.M.?

(e) If 15 repairs have occurred by in three hours, what is the probability that second of them occurred in the first hour?

5. Customers arrive at an automatic teller machine in accordance with a nonhomogeneous Poisson process. From 8 am until 10 am customers arrive at a rate of 5 an hour. Between 10 am and 2 pm the arrival rate steadily increases from 5 per hour at 10 am to 25 per hour at 2 am. From 2 pm to 8 pm the arrival rate steadily decreases from 25 per hour at 2 pm to 4 per hour at 8 pm. Between 8 pm and midnight the arrival rate is 3 an hour and from midnight to 8 am the arrival rate is 1 per hour.

(a) What is the probability distribution of the number of customers withdrawing money during a 24-hour period?

5. Continued

(b) What is the expected number of customers withdrawing money during a 24-hour period?

5. Continued

(c) The amounts the money withdrawn by the customers are independent and identically distributed random variables with a mean of KD100 and a standard deviation of KD125. Calculate the mean and the variance of the total withdrawal during 24 hours.

6. Extra Credit Problems:

i. What is a stochastic process? Give some examples for different types of stochastic processes and explain what do we mean by a Markov stochastic process?

ii. What are the special characteristics of the Discrete Time Markov Chain (DTMC or simply MC)?