Problem 1. 6 points

Consider the following linear program

$$
\begin{array}{lrcrcl}
\max\ z = & 3x_1 & + & 2x_2 & & \\
\text{subject to} & x_1 & + & 2x_2 & \leq & 6 \\
& 2x_1 & + & x_2 & \leq & 8 \\
& -x_1 & + & x_2 & \leq & 1 \\
& & & x_2 & \leq & 1 \\
& x_1, & & x_2 & \geq & 0
\end{array}
$$

(a) Is the point $x_1 = 0$, $x_2 = 0$ a basic feasible solution for the linear program (justify your answer by using the definition of a basic solution)?

(b) Is the point $x_1 = 1$, $x_2 = 1$ a basic feasible solution for the linear program (justify your answer by using the definition of a basic solution)?

(c) Represent the point $x_1 = 1$, $x_2 = 1$ as a convex combination of extreme points, plus, if applicable, a direction of unboundedness.

Problem 2. 6 points

Consider the following linear program

$$\begin{array}{lcccl} \min\ z = & 40x_1 & + & 36x_2 & \\ \text{subject to} & 5x_1 & + & 3x_2 & \geq 45 \\ & x_1 & & & \leq 8 \\ & & & x_2 & \leq 10 \\ & x_i & \geq & 0, & i = 1,\ 2 \end{array}$$

(a) Find the dual of the problem.

(b) Solve the dual problem.

(c) Use the dual optimal solution and the complementary slackness conditions to solve the primal problem.

(d) By how much the coefficient of x_2 in the objective function of the primal change before the current basis ceases to be optimal?

(e) Given that the optimal solution contains the variables x_1 and x_2 in the basis, by how much the right hand side of the first constraint of the primal change before the current basis ceases to be optimal?

Problem 3. 9 points

The following are final phase 1 simplex tableaux for different linear programming problems. In each problem a_1 and a_2 are the artificial variables for some given objective function.
For each problem, determine whether the problem is feasible (justify your answer).

(a)

	x_1	x_2	x_3	a_1	a_2	
$-\psi$	0	0	0	1	1	0
	3	0	1	-1	2	0
	2	1	0	0	1	5

(b)

	x_1	x_2	x_3	a_1	a_2	
$-\psi$	1	0	1	0	0	0
	3	1	0	0	1	2
	-1	0	-1	1	1	0

(c)

	x_1	x_2	x_3	a_1	a_2	
$-\psi$	0	1	2	0	0	-1
	0	1	-2	-3	1	1
	1	3	4	1	0	3

(e) In part (a) if the objective function is given by min $z = x_1 + x_2 + x_3$, construct the first tableau of phase 2 only.

Problem 4. 6 points

Consider the following linear program

$$\begin{array}{lrcrcl} \min\ z = & 2x_1 & + & 3x_2 & & \\ \text{subject to} & 3x_1 & - & 2x_2 & \geq & 4 \\ & x_1 & + & 2x_2 & \geq & 3 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

(a) Apply one iteration of the dual simplex method to the problem.

(b) Given that the optimal solution of the linear program is ~~given by~~ $x_1 = \frac{7}{4}$, and $x_2 = \frac{5}{8}$, apply parametric programming to the linear programming given that $\Delta c = (4, 1, 0, 0)'$.

Problem 5. 6 points

Apply at most two iterations of the revised bounded simplex method to solve the following linear program

$$
\begin{array}{lcrcrcl}
\max z & = & 8x_1 & + & 2x_2 & & \\
\text{subject to} & & 3x_1 & + & x_2 & \leq & 28 \\
 & & 5x_1 & + & 2x_2 & \leq & 42 \\
 & & x_1 & & & \leq & 8 \\
 & & & & x_2 & \leq & 8 \\
 & & x_1 & , & x_2 & \geq & 0
\end{array}
$$

with initial solution $x_1 = 0$ and $x_2 = 0$.

Problem 6. 7 points

Solve the following Linear program using the Affine Scaling Method starting from the point $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$.

$$
\begin{array}{llcccccccccc}
\max z & = & 4x_1 & + & 3x_2 & & & & & & \\
\text{subject to} & & -2x_1 & + & 2x_2 & + & x_3 & - & x_4 & = & 0 \\
& & x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \\
& & x_1 & , & x_2, & & x_3, & & x_4 & \geq & 0
\end{array}
$$

Perform at most two iterations