Problem 1. Consider the following linear programming problem.

$$\begin{array}{lrrcl} \max z & 8x_1+ & 2x_2 & & \\ \text{subject to} & 3x_1+ & 2x_2 & \leq & 20 \\ & 5x_1+ & 2x_2 & \leq & 22 \\ & x_1 & & \leq & 8 \\ & & x_2 & \leq & 8 \\ & x_1, & x_2, & \geq & 0 \end{array}$$

(a) Find the optimal solution using the graphical method.

(b) Using the representation theorem show that the solution found in (a) is indeed optimal.

Problem 2.

(a) Consider the linear programming problem:

$$\begin{array}{lrrrcl}
\max z & 4x_1+ & 3x_2+ & 6x_3 & & \\
\text{subject to} & 3x_1- & 4x_2- & 6x_3 & \leq & 18 \\
 & -2x_1- & x_2+ & 2x_3 & \leq & 12 \\
 & x_1+ & 3x_2+ & 2x_3 & \leq & 1 \\
 & x_1, & x_2, & x_3 & \geq & 0
\end{array}$$

Use the revised simplex method to solve this problem (perform at most two iterations).

(b) Use Bland's rule in part (a). Do you get the same solution after two iterations

Problem 3. Consider the linear programming problem:

$$\begin{array}{lrrrcl}
\max z & 4x_1+ & 3x_2+ & 6x_3 & & \\
\text{subject to} & 3x_1- & 4x_2- & 6x_3 & \leq & 18 \\
 & -2x_1- & x_2+ & 2x_3 & \leq & 12 \\
 & x_1+ & 3x_2+ & 2x_3 & \leq & 1 \\
 & x_1, & x_2, & x_3 & \geq & 0
\end{array}$$

The optimal solution to this problem is $z = 4$ at the point $x_1 = 1$, $x_2 = 0$, $x_3 = 0$, and the final tableau for the simplex method is:

	x_1	x_2	x_3	x_4	x_5	x_6	
	0	9	2	0	0	4	4
x_4	0	−13	−12	1	0	−3	15
x_5	0	5	6	0	1	2	14
x_1	1	3	2	0	0	1	1

(a) State the dual problem and find its optimal solution.

(b) Verify the complementary slackness theorem

(c) Find all values of Δc_2 such that the solution above remains optimal.

(d) Find all values of Δc_5 such that the solution above remains optimal.

(e) Find the optimal solution of the problem obtained by changing c_6 to 3.

(f) Suppose the final tableau is obtained from the initial tableau by multiplying by B^{-1}. Find B^{-1}.

(g) Find the optimal solution to the problem obtained by changing the constant term in the third constraint (b_3) from 1 to 5.

(h) Find the optimal solution to the problem obtained by changing the constant term in the third constraint (b_3) from 1 to 7.

(i) A further constraint $x_2 + x_3 \geq 1$ is added to the original problem. Use the dual simplex method to find an optimal solution to this new problem (if one exists).

Problem 4.

Use the revised bounded simplex method to solve the following linear programming problem (perform at most two iterations)

$$
\begin{array}{llll}
\max z & 8x_1 + 2x_2 & & \\
\text{subject to} & 3x_1 + 2x_2 & \leq & 28 \\
 & 5x_1 + 2x_2 & \leq & 42 \\
 & x_1 & \leq & 8 \\
 & x_2 & \leq & 8 \\
 & x_1, \ x_2, & \geq & 0
\end{array}
$$

Problem 5. Use the Affine Scaling Method to solve the following linear programming problem (perform at most two iterations) starting from the point $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$.

$$\begin{array}{lrrrrcl}
\max z & 4x_1+ & 3x_2+ & & x_4 & & \\
\text{subject to} & -2x_1+ & 2x_2+ & x_3- & x_4 & = & 0 \\
 & x_1+ & x_2+ & x_3+ & x_4 & = & 1 \\
 & x_1, & x_2, & x_3, & x_4 & \geq & 0
\end{array}$$