1. **(7 pts)** A graph is defined by the following adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

a) **(1.5 pts)** Draw the graph.

b) **(1.5 pts)** Write the incidence matrix.

c) **(1.5 pts)** Draw the geometric dual of the graph (if any).

d) Is the graph:

i) **(0.5 pt)** complete? why?

ii) **(0.5 pt)** connected? why?

iii) **(0.5 pt)** acyclic? why?

iv) **(0.5 pt)** a bi-partite? why?

v) **(0.5 pt)** a tree? why?

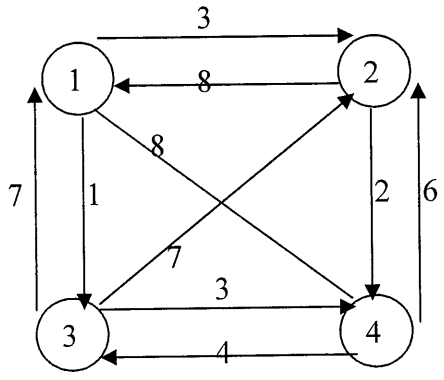
2. **(4 pts)** Consider an undirected network whose arc weights are given in the following table.

Node	1	2	3	4	5
1	0	10	6	3	9
2		0	16	2	13
3			0	3	8
4				0	15
5					0

a) **(2 pts)** Use Kruskal algorithm to find the maximum spanning tree.

b) **(2 pts)** Find the Steiner tree among nodes 1, 2, and 3.

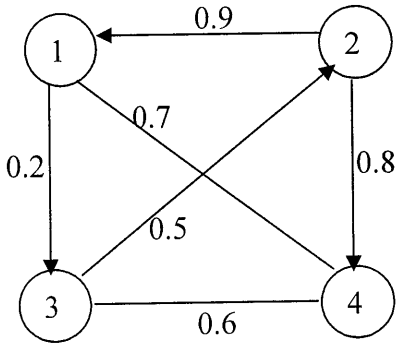
3. **(5 pts)** Consider the following network where the number beside each street shows the distance.



a) **(3 pts)** Use Floyd algorithm to find the matrix of shortest distances and the penultimate matrix for the network.

b) **(2 pts)** If a post office is located at node 1, what is the solution to the postman problem?

4. **(5 pts)** Consider the following network where the number beside each arc shows the reliability of the arc.

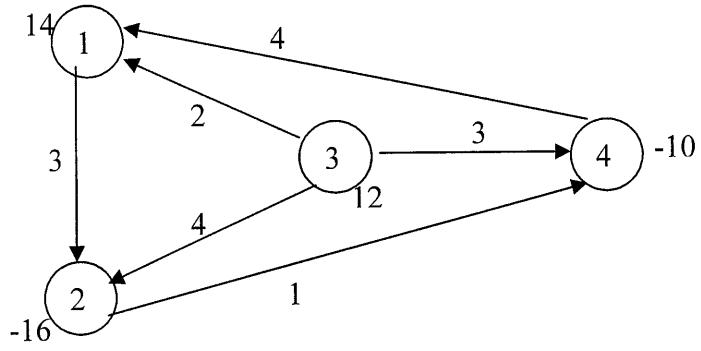


a) **(1.5 pts)** Convert this most reliable path problem into a shortest path problem by computing the appropriate arc weights.

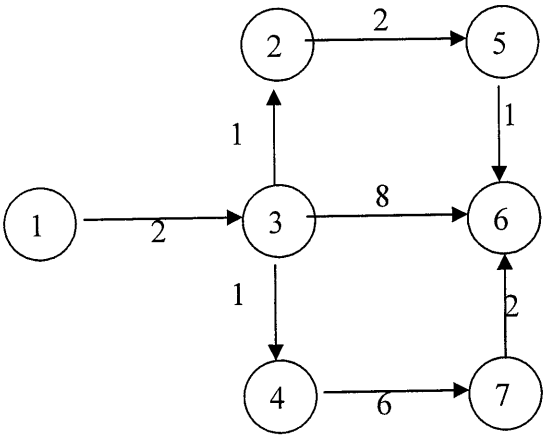
b) **(1.5 pts)** Use the network of part (a) to present an LP that can be used to find the most reliable path between node 1 and node 4.

c) **(2 pts)** Apply Ford algorithm to the network of part (a) to find to the most reliable paths from node 1 to other nodes.

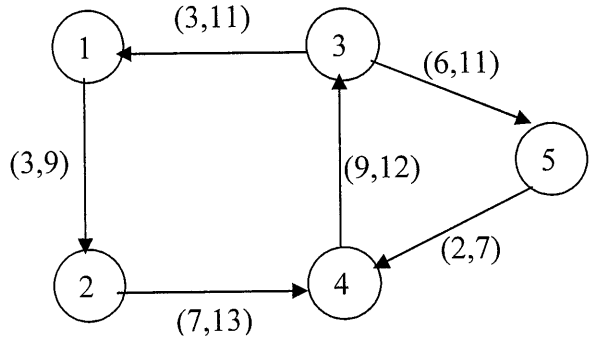
5. **(3 pts)** Consider the following network where the number beside each node shows the supply/demand, and the number beside each arc gives the cost per unit of flow. Given the feasible flow $f_{12} = 14, f_{32} = 12, f_{24} = 10$, and all other $f_{ij} = 0$, use the network simplex method to solve this minimum cost flow problem.



6. **(4 pts)** Consider the following network in which the number beside each arc shows the travel time (in minutes). Suppose that at each node there is a delay of 1 minute for turning right, 2 minutes for turning left, and no delay for going straight. Construct a network without turn delays so that it can be used to find shortest paths from node 1 to other nodes.



7. **(7 pts)** Consider the following maximum flow network where the numbers beside each arc show, respectively, the current flow and the capacity of the arc.



a) **(0.5 pt)** Identify the source and sink of the network. Justify your answer.

b) **(0.5 pt)** What is the current flow between the source and the sink? Why?

c) **(2 pts)** Present a linear programming formulation to find the maximum additional flow from the source to the sink.

d) **(2 pts)** Use the method of flow augmenting path to compute the maximum additional flow from the source to the sink.

e) **(2 pts)** Use the maximum flow algorithm to compute the maximum additional flow from the source to the sink.

8. **(5 pts)** There are persons A, B and C, and four jobs 1, 2, 3, and 4. Due to applicant and job specifications, person A can be assigned to either job 1 or 2; person B can be assigned to either job 2 or 3; person C can be assigned to either job 3 or 4.

a) **(2 pts)** Model this problem as a matching network problem.

b) **(2 pts)** Use the network of part (a) to find the maximal matching.

c) **(1 pts)** Use Hall's marriage theorem to show that there is a complete matching.