1. **(6 pts)** A graph is defined by the following incidence matrix.

$$\begin{bmatrix} +1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & +1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & +1 & -1 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & +1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

a) **(1.5 pts)** Draw the graph.

b) **(1.5 pts)** Write the adjacency matrix.

c) **(1.5 pts)** Draw the geometric dual of the graph (if any).

d) Is the graph:

i) **(0.5 pt)** complete? why?

ii) **(0.5 pt)** connected? why?

iii) **(0.5 pt)** acyclic? why?

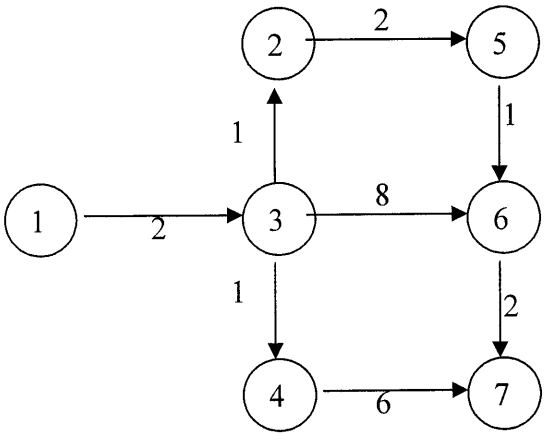
2. **(5 pts)** Consider an undirected network whose arc weights are given in the following table.

Node	1	2	3	4	5
1	0	9	5	2	8
2		0	15	1	12
3			0	2	7
4				0	14
5					0

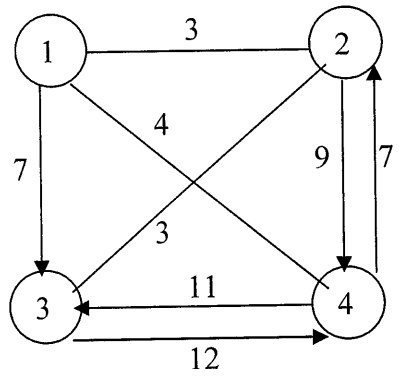
a) **(2.5 pts)** Use Prim algorithm to find the maximum spanning tree.

b) **(2.5 pts)** Find the Steiner tree among nodes 1, 2, and 3.

3. **(4 pts)** Consider the following network in which the number beside each arc shows the travel time (in minutes). Suppose that at each node a delay of 1 minute incurs for turning either right or left, while there is no delay for going straight. Reformulate this network as a shortest path problem without turn delays.



4. **(6 pts)** Consider the following network where the number beside each arc shows the distance.



a) **(3 pts)** Apply the Floyd algorithm to find the matrix of shortest distances and the penultimate matrix for the network.

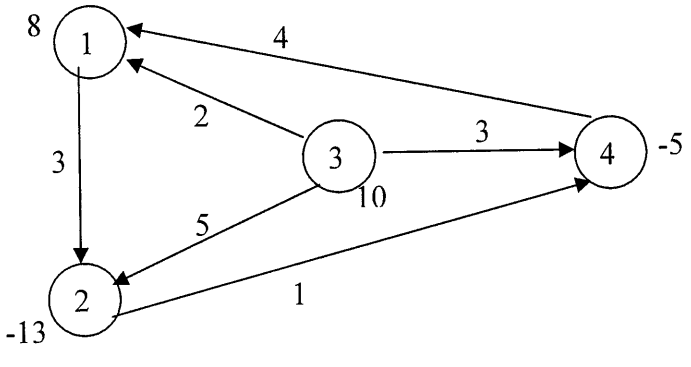
b) **(3 pts)** If a post office is located at node 3, what is the solution to the Chinese postman problem?

5. **(4 pts)** A company produces a product during 4 months of a year. The demand, unit variable cost, minimum and maximum production capacities, unit inventory cost, and maximum storage capacity for the product during each month are as follow:

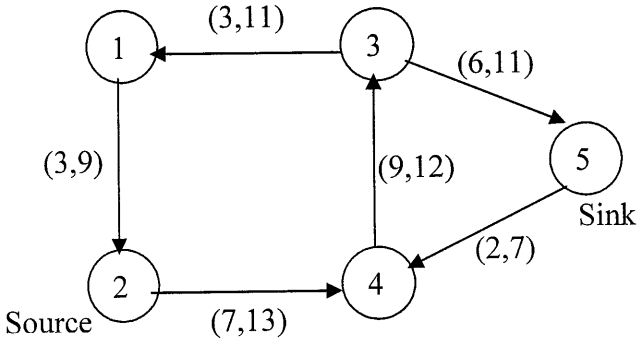
Month	1	2	3	4
Demand:	120	180	140	250
Variable cost($):	60	80	70	90
Minimum cap.:	0	10	5	10
Maximum cap.:	100	80	-	100
Inventory cost($):	10	12	5	8
Storage cap.:	30	40	50	20

The company want to find the production level during each month to minimize the total cost subject to the given demand and capacity constraints. Construct a minimum-cost network flow for this problem in which flows are the monthly level of production.

6. **(3 pts)** Consider the following network where the number beside each node shows the supply/demand, and the number beside each arc gives the cost per unit of flow. Given the feasible flow $f_{12} = 13, f_{31} = 5, f_{34} = 5$, and all other $f_{ij} = 0$, use the network simplex method to solve this minimum cost flow problem.



7. **(7 pts)** Consider the following network. The numbers beside each arc show, respectively, the current flow and the capacity of the arc.



a) **(1 pt)** What is the current flow between the source and the sink?

b) **(3 pts)** Use the maximum flow algorithm to compute the maximum additional flow that can be sent from the source to the sink.

c) **(3 pts)** Present a linear programming formulation to find the maximum flow that can be send from the source to the sink.

8. **(5 pts)** There are three machines A, B and C, and three locations 1, 2, and 3. Due to machine and location specifications, machine A can be assigned to either location 1 or location 2; machine B can be assigned to either location 2 or location 3; machine C can be assigned to either location 1 or location 3.

a) **(2 pts)** Model this problem as a matching network problem.

b) **(2 pts)** Use the network of part (a) to find the maximal matching.

c) **(1 pts)** Is the matching of part (b) a complete matching? Why?