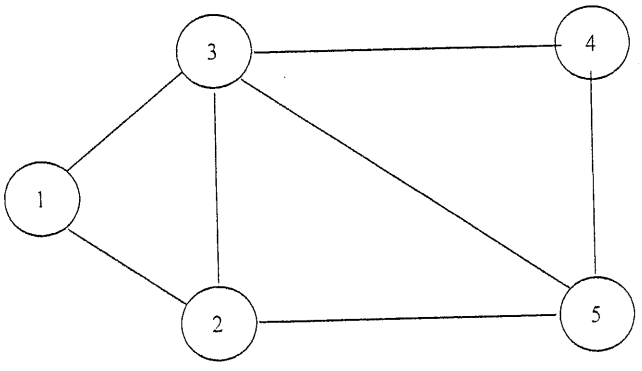
Problem 1.



1. Find 3 different spanning trees of the above graph.

2. Write the incidence matrix.

3. Write the adjacency matrix.

4. Is the graph complete?

5. Is the graph connected? Why?

6. Is the graph planar? If yes construct its geometric dual.

Problem 2 A pipeline system must be built to connect 5 refineries with a port facility that receives imported crude oil. The cost of building the pipeline between any two points is 280 KD per mine plus a 1000 KD cost for each segment. The distance between all pairs of points are given in the following table.

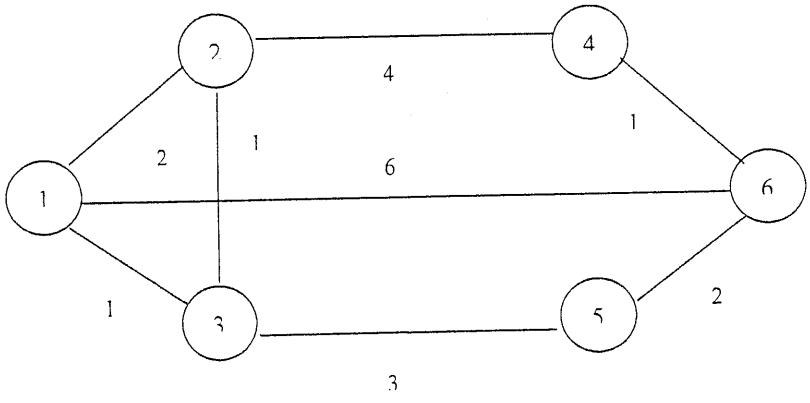
	P	R1	R2	R3	R4	R5
Port	0	5	6	8	2	9
Refinery 1		0	4	10	5	6
Refinery 2			0	11	8	9
Refinery 3				0	10	6
Refinery 4					0	5
Refinery 5						0

1. Find the least cost pipeline. (**Hint**: *Use Spanning Tree*)

2. If for technical reasons no pipes can be installed between P and 5 and 1 and 2, what is the minimum cost pipeline which can be built to connect the 5 refineries with the port?

Find the Steiner Tree among refineries 1, 2, and 3 with the minimum cost.

Problem 3.

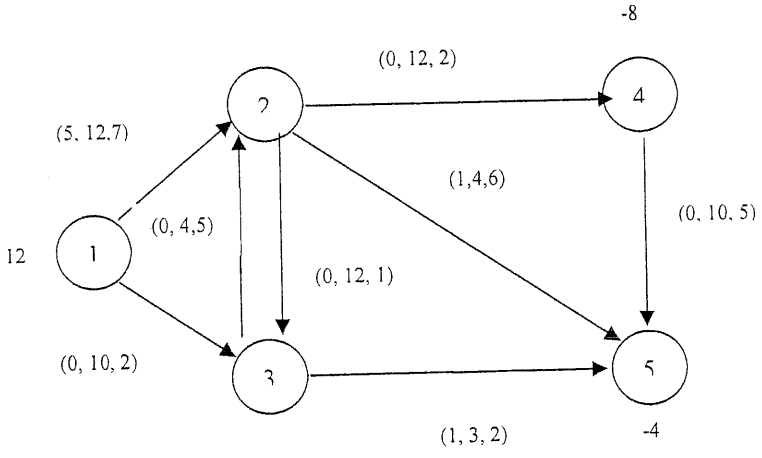


1. Use Floyd's Algorithm.to find the shortest distance matrix and penultimate matrix for the network.

2. Use the matrices in part 1 to find the shortest paths and their distances from vertex 1 to vertex 5 and from vertex 1 to vertex 6.

3. Suppose a post office is located in vertex 1. A postman leaves the post office in the morning to deliver mail along the streets (arcs) of the network, and return back to the post office. What advise would you give to the postman with regard to the best strategy that he should adopt.

Problem 4.

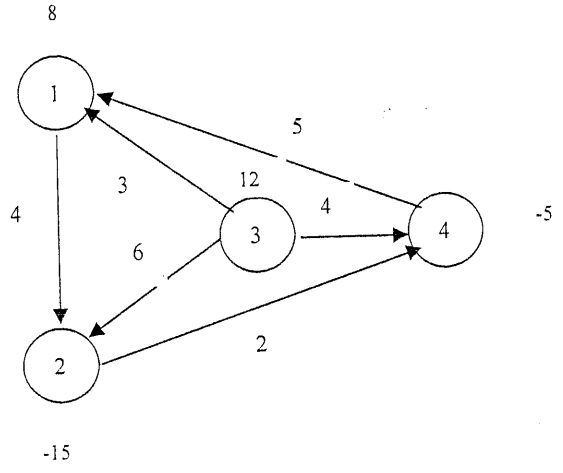


1. Apply the necessary transformations to remove lower and upper bounds on arc flows and construct the resulting network.

2. Set-up a transportation network for part 1.

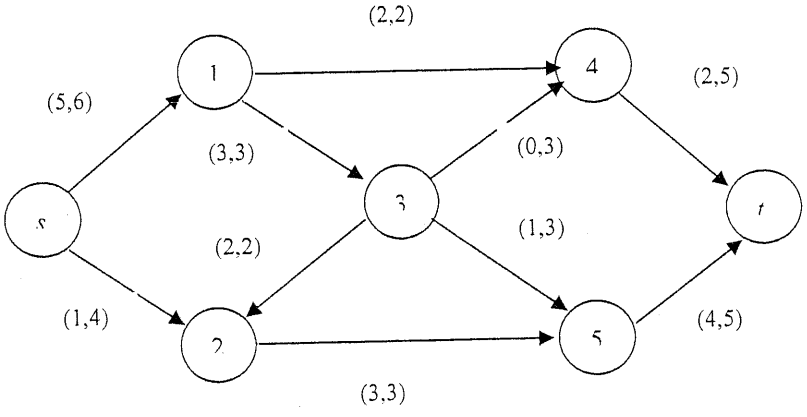
Problem 5

Consider the following min-cost flow network where the number beside each node shows the supply/demand, and the number beside each arc gives the cost per unit of flow. A feasible flow $f_{12}=15$, $f_{31}=7$, $f_{34}=5$, and all other $f_{ij}=0$ is given.



Use the network simplex method to find the min-cost flow.

Problem 6 Consider the problem of finding the maximum flow for the network below from s to t, where the notation (x, y) on the arcs refers to the flow x and the capacity y.



1. Check that the given flow is feasible.

2. Is the initial flow optimal? (Justify your answer).

3. Find the maximum flow.

4. What is the minimum cut? Verify that the capacity of this cut is equal to the maximum flow.

Problem 7.

A machine shop posses six different drilling machines. On a certain day, five jobs that need drilling arrive. The number of person-hours required to perform each job on each of the machines is given below. Find the best way to assign each job to a different machine.

	Job				
Machine	A	B	C	D	E
1	5	7	6	4	9
2	8	10	3	4	7
3	6	11	5	4	7
4	5	8	7	3	9
5	3	6	4	2	7
6	3	7	5	3	7