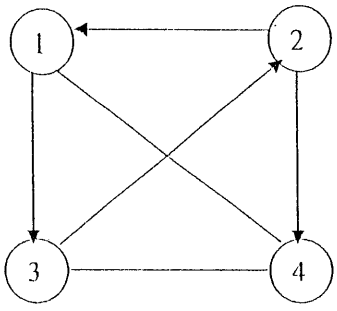
1. **(6 pts)** Consider the following graph.



a) **(1 pt)** Write the incidence matrix.

b) **(1 pt)** Write the adjacency matrix.

c) **(1 pt)** Write the adjacency list.

d) Is this graph:

i) **(0.5 pt)** a bi-partite? why?

ii) **(0.5 pt)** complete? why?

iii) **(0.5 pt)** connected? why?

iv) **(0.5 pt)** acyclic? why?

v) **(1 pt)** a planar? Why? If a planar, draw its geometric dual.

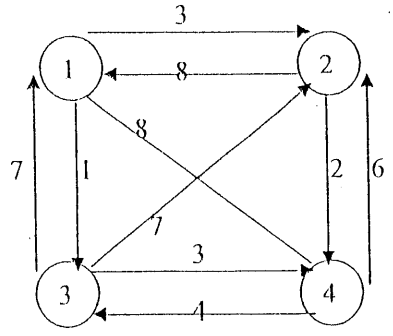
2. **(5 pts)** A company wants to distribute natural gas among 5 towns through a system of pipelines. The costs (in millions of $) for installing gas pipes between pairs of towns are given in the following table.

Town	1	2	3	4	5
1	0	10	8	3	9
2		0	16	2	13
3			0	3	8
4				0	15
5					0

a) **(3 pts)** How can gas be distributed among the towns with the minimum pipe cost? What is this cost? (Hint: Use Prim algorithm.)

b) **(2 pts)** Find the Steiner tree for distributing gas among towns 1, 2, and 3 with the minimum pipe cost.

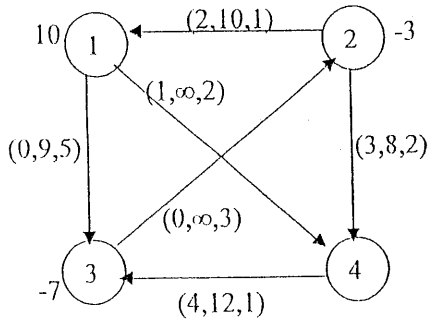
3. **(6 pts)** Consider the following network of streets among four districts of a town. The number beside each street shows the distance.



a) **(4 pts)** Use Floyd algorithm to find the matrix of shortest distances and the penultimate matrix for the network.

b) **(2 pts)** Suppose that a post office is located in district 1. Each day a mailman must carry mails from the post office for delivery along the 10 streets connecting the four districts, and then return back to the post office. Could you advise the mailman how to deliver the mails to minimize his daily travel distance?

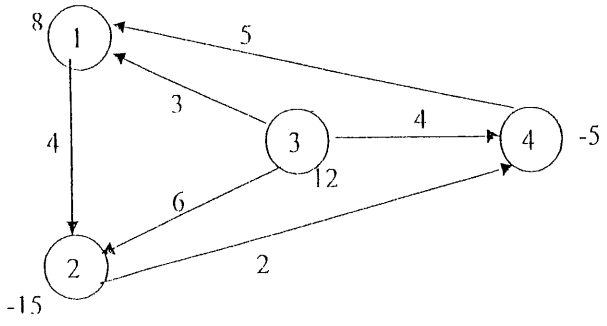
4. **(6 pts)** Consider the following min-cost flow problem. The numbers beside each arc show the lower and upper bound capacities on flow and the unit cost. The number beside each node gives the supply/demand.



a) **(4 pts)** Remove the lower and upper bound capacities on flows and give the resulting network.

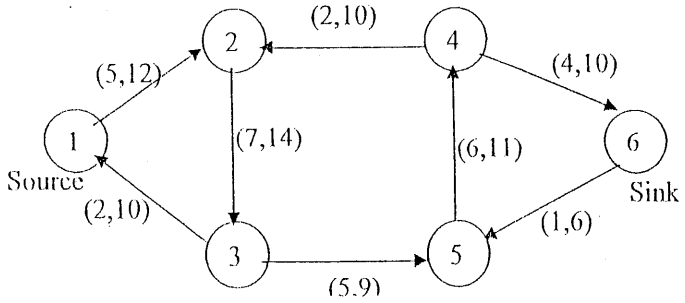
b) **(2 pts)** Set up the transportation/transshipment network for the network of part (a).

5. **(4 pts)** Consider the following min-cost flow network where the number beside each node shows the supply/demand, and the number beside each arc gives the cost per unit of flow. A feasible flow $f_{12} = 15$, $f_{31} = 7$, $f_{34} = 5$, and all other $f_{ij} = 0$ is given.



Use the network simplex method to find the min-cost flow.

6. **(7 pts)** Consider the following communication network. The numbers beside each arc (telephone line) show the current number of telephone calls and the call capacity of the arc.



a) **(1 pt)** What is the current number of calls between the source and the sink?

b) **(3 pts)** Use the maximum flow algorithm to compute the maximum additional number of calls that can be made from the source to the sink.

c) **(3 pts)** Present a linear programming formulation to find the maximum number of calls that can be made from the source to the sink.

7. **(6 pts)** There are three applicants A, B, and C, and four jobs 1, 2, 3, and 4. Due to each applicant qualification and job requirements, applicant A can be assigned to either job 1 or job 2; applicant B can be assigned to one of jobs 1, 2, and 3; and applicant C can be assigned to either job 2 or job 4.

a) **(1 pt)** Model this problem as a matching bi-partite.

b) **(2 pts)** Model this problem as a matching network problem.

c) **(2 pts)** Use the network of part (b) to find the maximal matching.

d) **(1 pts)** Is the matching of part (c) a complete matching? Why?