Problem 1. A barbershop has chairs for two waiting customers plus one getting a haircut. Customer who arrive when all chairs are occupied will have to stand. Customers arrive at a Poisson process of one every 15 minutes. The barber takes an average of 10 minutes to trim each customer, the actual times having an exponential density.

1. Find the average number of customers in the barber shop.

2. What percentage of the time the barber is busy.

3. What is the probability that an arriving customer will find no seat available.

Problem 2. A bank has two tellers working. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distributions for both deposits and withdrawals are exponential with mean service time 3 minutes per customer. Depositors are found to arrive in a Poisson process throughout the day with mean arrival rate 16 per hour. Withdrawals also arrive in a Poisson process with mean arrival rate 14 per hour.

1. Find the expected waiting time for depositors and withdrawals before being served?

2. If the system is redesigned so that each teller can handle both withdrawals and deposits, what would be the effect on the expected waiting time found in 1 for both depositors and withdrawals?

Problem 3. Consider an $M/M/c$ Queuing System where customers waiting for some time in the Queue decides to leave the system. We assume that in a short interval $(t, t+dt)$

$$P\{\text{ a customer waiting at } t \text{ leaves the system during } (t, t+dt) | Q(t) = n\}$$
$$= \nu dt + o(dt) \qquad \text{if } n \geq c$$
$$= 0 \qquad \text{if } 0 \leq n < c$$

Show that at the steady state the number of customers in the system is given by

$$p_n = \begin{cases} \frac{p_0}{n!}\left(\frac{\lambda}{\mu}\right)^n & (0 \leq n \leq c) \\ p_c \prod_{i=1}^{n-c} \left(\frac{\lambda}{c\mu + i\nu}\right) & n \geq c \end{cases}$$

Problem 4 A grocery store has a single checkout counter attended by a cashier who also functions as the bagger when the store is too busy. Customers arrive at the checkout counter according to a Poisson process, at a mean rate of 30 per hour. The time required for the cashier to total a customer's purchases is exponentially distributed with a mean of 2 min. Whenever there are three or more customers at the counter (including the customer in service), second employee of the store is instructed to assist the cashier as a bagger. When the two employees work together, the service time for a customer remains exponentially distributed, but with a mean of 1 min.

1. Determine the average number of customers in the queue including the one being served.

2. Determine the waiting time in the system.

3. Determine the waiting time in the queue.

4. Rework parts 1-3 if you assume that second employee comes as a separate equally efficient cashier-bagger, working in parallel with the first. Whenever, only two customers remain, the momentarily free employee leaves the check-out counter, to return whenever the state again reaches 3. Would this arrangement be preferable from the customers' point of view?

Problem 5. Jobs arrive at a machine according to a Poisson process with a rate of 5 jobs per hour. Each job consists of 2 tasks. Each task has an exponentially distributed processing time with a mean of 1.5 minute. Jobs are processes in order of arrival.

1. Determine the number of uncompleted tasks in the system.

2. Determine the distribution of the number of jobs in the system.

Problem 6. Customers arrive at a checkout counter in a supermarket accotding to a Poisson process of rate $\lambda = 1$ customer per minute. The checkout counter cab be operated with or without a bagger. The checkout times for customers are random. With a bagger the mean checkout time is 30 seconds, while without a bagger this mean time increases to 50 seconds. In both cases, the standard deviation of service time is 10 seconds. Compare the mean queue lengths with and without a bagger.