

Q1. (10 marks) A physical therapist wished to compare three methods for teaching patients to use a certain prosthetic device. He felt that the rate of learning would be different for patients of different ages and wished to design an experiment in which the influence of age could be taken into account. So he used a randomized complete block design and obtained the following data.

Table : Time (in days) required for learning use of device

Age group	<u>Teaching method</u>		
	A	B	C
Under 20	7	9	10
20 to 29	8	9	10
30 to 39	9	9	12
40 to 49	10	9	12
50 and over	11	12	14

Provide a **complete analysis** of the data and your conclusions (including the ANOVA table).

Q2. (10 marks) In a study of length of time spent on individual home visits by public health nurses, data were reported on length of home visit, in minutes, by a sample of 32. A record was made also of each nurse's age and the type of illness of each patient visited. The data obtained are given in the following table.

Table : Length of home visits (in minutes)

Factor A level	Factor B (nurses age group) level			
	1(age 20 – 29)	2(age 30 – 39)	3(age 40 – 49)	4(age 50+)
1 (Cardiac)	20	25	24	20
	25	30	28	31
2 (Cancer)	30	30	39	40
	45	29	42	45
3 (C.V.A.)	31	32	41	42
	30	35	45	50
4 (T.B)	20	23	24	29
	21	25	25	30

(a) **State** and **explain** a potentially suitable statistical model for this data set. (2)

(b) **State** the least squares estimates (formula) of the parameters in the model and their numerical values that you get. State also the ANOVA table. (5)

(c) Does the mean length of the home visit differ among the different age groups of nurses? Justify your answer. (1)

(d) Does the type of patient affect the mean length of home visit? Justify your answer. (1)

(e) Is there interaction between nurse's age and type of patient? Justify your answer. (1)

Q3. (12 marks)

(a) Consider a 2^3 factorial experiment involving three factors A, B and C each at two levels with each “factor combination” (treatment) being applied to n units.

(i) **Write** all the treatments using “ \pm notation” and “geometric 0 and 1 notation”. (1)

(ii) **State** the formula for effect A, interactions AB & ABC using algebraic notation. (3)

(iii) **Interpret** A, AB and ABC graphically drawing a separate graph for each. (3)

(b) A 2^3 factorial experiment involving factors A, B and C was replicated 2 times and the results were as follows:

TREATMENT	REPLICATE 1	REPLICATE 2
(1)	45	43
a	71	100
b	48	45
ab	65	104
c	68	75
ac	60	86
bc	80	70
abc	65	96

Provide a complete analysis of the data. (5)

Q4. (9 marks) Consider the following random effects model for a single factor experiment $y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ ($i = 1, \dots, a; j = 1, \dots, n_i$).

(i) **Explain** the various terms in the model (1)

(ii) In what situations is the model better than a fixed effects model? (1)

(iii) **State** values of $\text{var}(y_{ii})$, $\text{corr}(y_{ij}, y_{ik})(j \neq k)$, $\text{corr}(y_{ij}, y_{kl})(i \neq k)$. (2)

(iv) **Define** SS_T , SS_{tr} , MS_{tr} , SS_E & MS_E and **show** that $SS_T \equiv SS_{tr} + SS_E$ (3)

(v) **State** $E(MS_{tr})$ & $E(MS_E)$ & hence an unbiased estimate of $\text{var}(\tau_i)$. (2)

Q5. (9 marks) Several ovens in a metal working shop are used to heat metal specimens. All the ovens are supposed to operate at the same temperature, although it is suspected that this may not be true. Three ovens are selected at random and their temperatures on successive heats are noted. The results are as follows :

Oven	Temperature					
1	491.50	498.30	498.10	493.50	493.60	
2	488.50	484.65	479.90	477.35		
3	490.10	484.80	488.25	473.00	471.85	478.65

(a) Would you use a fixed-effects model or a random-effects model to analyze the above data? Justify your answer and get the ANOVA table to answer the following. (2)

(b) Is there significant variation in temperature between ovens? (1)

(c) Assuming a random effects model, estimate the components of variance for this model and also the intra-class correlation, (2)

(d) Obtain a 95% C.I. for the intra-class correlation, (1)

(e) Analyze the residuals and draw conclusions about model adequacy? (3)