QUESTION 1: [*6 Points*]

a. Consider the *linear congruential generator* (LCG) defined by $m = 100$, $a = 17$, $c = 43$, and $Z_0 = 27$. Generate a sequence of three Uniform(0, 1) pseudo-random numbers.

b. Without actually computing any Z_i's, determine whether or not the *linear congruential generator* (LCG) in part (a) has full cycle.

QUESTION 2: [*8 Points*]

a. At the end of any day, the number of shipments on loading dock of the KOC is either 0, 1, or 2 with observed relative frequency of occurrence of 0.5, 0.3, and 0.2, respectively. Internal operations research consultants have been asked to develop a model to improve the efficiency of the loading and hauling operations; as part of this model, they will need to be able to generate values, X, to represent the number of shipments on loading dock at the end of each day. The consultants decide to model X as a discrete random variable with the probability mass function (pmf), $p(x)$, given in the following table. Use the Inverse Transform Technique to generate three realizations of the random variable X. Let the pseudo-random U[0, 1] inputs be $U_1 = 0.3567$, $U_2 = 0.736$, and $U_3 = 0.6721$.

x	0	1	2
$p(x)$	0.5	0.3	0.2

b. Times to failure for an automated production process have been found to be randomly distributed with an exponential distribution with probability density function (pdf)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

i. Construct an inverse algorithm for generating X.

ii. Let the pseudo-random U[0, 1] input be U_1 = 0.25 and $\lambda = 1$, generate a realization of the random variable X

QUESTION 3: [*12 Points*]

City buses arrive to the maintenance facility with exponential interarrival times with mean 2 hours. The facility consists of a single inspection station and two identical repair stations; see Figure 1. Every bus is inspected, and inspection times are distributed uniformly between 15 minutes and 1.05 hours; the inspection station is fed by a single FIFO queue. Historically, 30% of the busses have been found during inspection to need some repair. The two parallel repair stations are fed by a single FIFO queue, and repairs are distributed uniformly between 2.1 hours and 4.5 hours. Create a simulation model of this system.

a. Run the simulation for 160 hours and compute :
   i. The average delay in each queue,
   ii. The average length of each queue,
   iii. The utilization of the inspection station and the utilization of the repair station.

b. Replicate the simulation 10 times and compute 95% confidence intervals for the above measures of performance.

c. About how many replications would be required to bring the half width of a 95% confidence interval for the expected average queue length of the inspection station to 0.01 buses.

d. Run the simulation with the number of replications obtained in part **c**, and compute a 95% confidence interval on the expected average length of the inspection station queue.

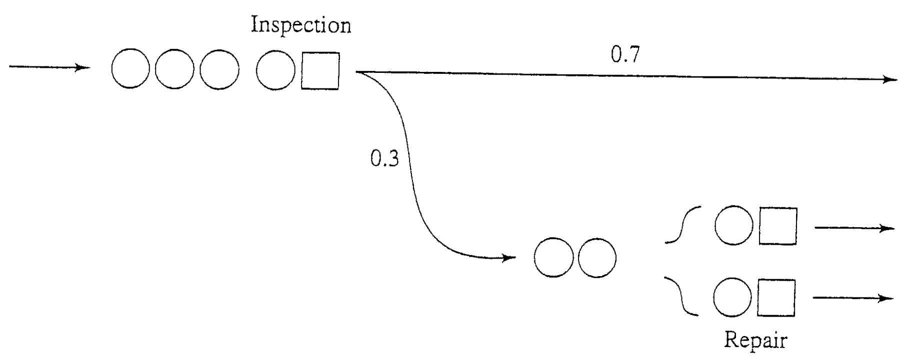

FIGURE 1
A bus maintenance depot.

QUESTION 4: [8 points]

In Question 3, Run the simulation for 10 replications. Assume all times are the same as before, and observe statistics on the average time in the system under each of the following scenarios:

i. Assign 1 server for the inspection station and two identical servers for the repair station (This scenario is the same as in question 3).
ii. Assign two identical servers for the inspection station and 1 server for the repair station .

Which of the two scenarios above is best ? Viewing this as terminating simulation, address this question in a statistically valid way.

QUESTION 5: [*6 Points*]

Parts arrive at a single workstation system according to an exponential interarrival distribution with mean 2 seconds. Upon arrival the parts are processed. The processing time distribution is exponential with mean 1.8 seconds. Suppose we would like to simulate the system to obtain a point estimate and a 95% confidence interval for the ***steady-state*** mean delay in the queue. In order to use the replication/deletion approach, you must first eliminate the transient phase data. You must also use multiple simulation runs to generate *independent* and identically distributed random variables from which to compute the point estimate and the confidence interval.

[Use 10 replications to determine the warming up period, and 30 replications to estimate the steady state performance measure]