

[1] (7 points)

For the following IP problem

$$\text{Max } Z = 7X_1 + 10X_2$$

s.t.

$$\begin{aligned} -X_1 + 3X_2 + X_3 &= 6 \\ 7X_1 + X_2 + X_4 &= 35 \end{aligned}$$

$$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, X_4 \geq 0 \text{ and integer}$$

The optimal LP tableau is given below

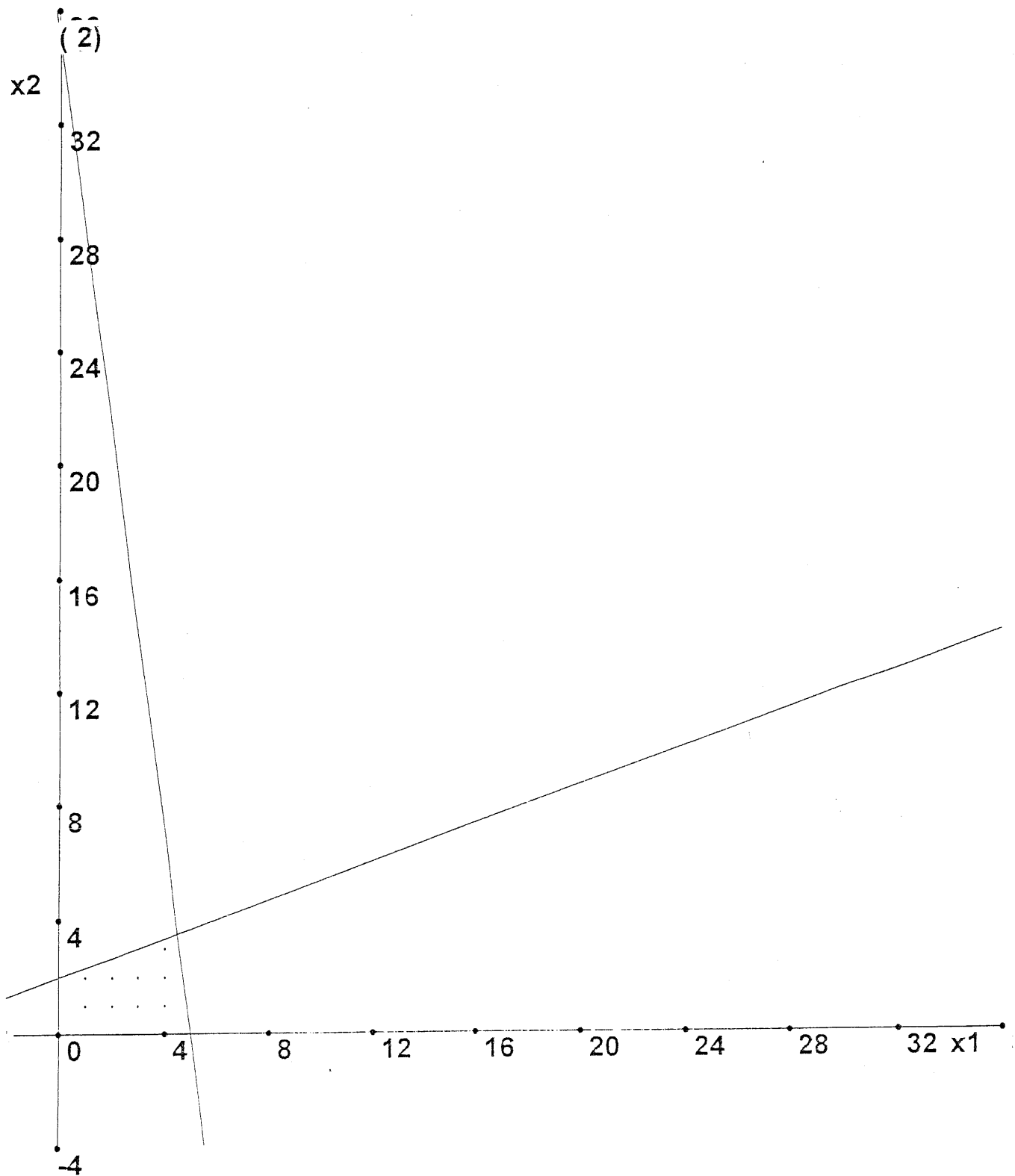
Basic	X_1	X_2	X_3	X_4	Solution
Z	0	0	$63/22$	$31/22$	$133/2$
X_2	0	1	$7/22$	$1/22$	$7/2$
X_1	1	0	$-1/22$	$3/22$	$9/2$

(a) Construct the cut associated with the X_2 row.

(b) Add the cut obtained in (a) to the above tableau and determine the entering and leaving variables.

Basic	X_1	X_2	X_3	X_4		Solution
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X_2	0	1	$7/22$	$1/22$		$7/2$
X_1	1	0	$-1/22$	$3/22$		$9/2$

(c) Using the feasible space for the above problem, draw the cut obtained in (a).



[2] (9 points)

Consider the design of an electronic device consisting of 3 main components. The three components are arranged in series so that the failure of one component will cause the failure of the entire device. The design calls for using one or two standby units. The following table charts the reliability, R, and cost, C.

No of parallel units	Component 1		Component 2		Component 3	
	R1	C1	R2	C2	R3	C3
1	0.5	2	0.6	2	0.7	3
2	0.7	3	0.8	4	0.8	5
3	0.9	4	0.9	5	0.9	6

The total capital available for the construction of the device is 10,000. How the device should be constructed to maximize the reliability.

(a) For the above problem, define the following

Stage i :

State x_i :

Alternative m_i :

$f_i(x_i)$:

(b) Use the given calculation for stage 3 to obtain the optimal solution for the problem.

Stage 3.

x_3	$f_3(x_3) = \max_{m_3=1,2,3} R_3(m_3)$			Optimum solution	
	$m_3=1$	$m_3=2$	$m_3=3$	$f_3(x_3)$	m_3^*
3	0.7	---	---	0.7	1
4	0.7	---	---	0.7	1
5	0.7	0.8	---	0.8	2
6	0.7	0.8	0.9	0.9	3

Stage 2.

Stage 1.

Optimal Solution

[3] (7 points)

Kuwait Dairy Danish Company (KDD) currently replenishes its stock of a certain item by ordering enough supply to cover a 1-month demand. The annual demand of the item is 1500 units. It is estimated that it costs 20 KD every time an order is placed. The holding cost per unit inventory per day is 0.067 and no shortage is allowed.

A. Determine the cost of the current annual inventory policy.

B. Determine the optimal order quantity, the time between orders and the reorder point given a lead time of 15 days.

C. Determine the difference in annual inventory costs between the optimal policy and the current policy of ordering a 1-month supply 12 times a year.

[4] (7 points)

The owner of a newsstand wants to determine the number of newspapers that must be stocked at the start of each day. It costs 45 fils to buy a copy, and the owner sells it for 100 fils. Newspapers left at the end of the day are recycled for an income of 10 fils a copy. How many copies should the owner stock, assuming that the demand for the day can be described as

- A. Normal distribution with mean 500 copies and standard deviation of 50 copies.
- B. Discrete pdf, $f(D)$, defined as

D	350	400	500	530	560
$f(D)$	0.1	0.2	0.4	0.2	0.1

[5] (7 points)

Alola car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of one car every 15 minutes. Cars wait in the facility's parking lot if the bay is busy. The time for washing and cleaning a car is exponential, with a mean of 12 minutes. Cars that cannot park in the lot can wait in the street bordering the wash facility.

(Tora output for the above problem is attached)

A. For the above queuing model fills in the blanks below

(...../...../.....) : (...../...../.....) $\lambda = \dots\dots\dots$, $\mu = \dots\dots\dots$

B. Determine the probability that an arriving car must wait in the parking lot prior to entering the wash bay.

C. If there are seven parking spaces, determine the probability that an arriving car will find an empty parking space.

D. Determine the percent utilization of the wash bay facility.

E. How many parking spaces should be provided so that an arriving car may find a parking space 99% of the time?

QUEUEING OUTPUT ANALYSIS

Title: p5
 Scenario

Lambda =	Mu =	
Lambda eff =	Rho/c =	
Ls = 4.00000	Lq = 3.20000	
Ws = 1.00000	Wq = 0.80000	

n	Probability, pn	Cumulative, Pn	n	Probability, pn	Cumulative, Pn
0	0.20000	0.20000	23	0.00118	0.99528
1	0.16000	0.36000	24	0.00094	0.99622
2	0.12800	0.48800	25	0.00076	0.99698
3	0.10240	0.59040	26	0.00060	0.99758
4	0.08192	0.67232	27	0.00048	0.99807
5	0.06554	0.73786	28	0.00039	0.99845
6	0.05243	0.79028	29	0.00031	0.99876
7	0.04194	0.83223	30	0.00025	0.99901
8	0.03355	0.86578	31	0.00020	0.99921
9	0.02684	0.89263	32	0.00016	0.99937
10	0.02147	0.91410	33	0.00013	0.99949
11	0.01718	0.93128	34	0.00010	0.99959
12	0.01374	0.94502	35	0.00008	0.99968
13	0.01100	0.95602	36	0.00006	0.99974
14	0.00880	0.96482	37	0.00005	0.99979
15	0.00704	0.97185	38	0.00004	0.99983
16	0.00563	0.97748	39	0.00003	0.99987
17	0.00450	0.98199	40	0.00003	0.99989
18	0.00360	0.98559	41	0.00002	0.99991
19	0.00288	0.98847	42	0.00002	0.99993
20	0.00231	0.99078	43	0.00001	0.99995
21	0.00184	0.99262	44	0.00001	0.99996
22	0.00148	0.99410			

[6] (7 points)

An airline ticket office has two ticket agents answering incoming phone calls for flight reservations. In addition, one caller can be put on hold until one of the agents is available to take the call. If all three phone lines (both agent lines and the hold line) are busy, a potential customer gets a busy signal, and it is assumed that the call goes to another ticket office and that the business is lost. The calls and attempted calls occur at a Poisson rate of 15 per hour. The length of a telephone conversation has an exponential distribution with a mean of 4 minutes. The airline ticket office open 24 hours a day.

(Tora output for the above problem is attached)

A. For the above queuing model fills in the blanks below

(...../...../.....) : (...../...../.....) $\lambda = \dots\dots\dots$, $\mu = \dots\dots\dots$

B. Find the probability that :

(i) A caller will get to talk to an agent immediately

(ii) The caller will be put on hold

(iii) The caller will get a busy signal

C. Determine the rate per day of lost customers

QUEUEING OUTPUT ANALYSIS

Title: p4

Ls =	1.00000	Lq =	0.09091
Ws =	0.07333	Wq =	0.00667

n	Probability, pn	Cumulative, Pn	n	Probability, pn	Cumulative, Pn
0	0.36364	0.36364	2	0.18182	0.90909
1	0.36364	0.72727	3	0.09091	1.00000

[7] (6 points)

Find the optimal solution for the following unconstrained nonlinear programming problem

$$\begin{aligned} \text{Max } f(x) &= -x_1^2 - 6x_2^2 - 4x_1 + 8x_2 + 143 \\ \text{s.t } x &\in \mathbb{R}^2 \end{aligned}$$

[7] (6 points)

Find the optimal solution for the following unconstrained nonlinear programming problem

$$\begin{aligned} \text{Max } f(x_1, x_2) &= -x_1^2 - 6x_2^2 - 4x_1 + 8x_2 + 143 \\ \text{s.t } x &\in \mathbb{R}^2 \end{aligned}$$