1. **(7 pts)** Consider the following ILP.

$$\text{Min } Z = 6x_1 + 8x_2$$

s.t.

$$3x_1 + x_2 \geq 4$$
$$x_1 + 2x_2 \geq 4$$
$$x_1, x_2 \geq 0; \text{ both } x_1 \text{ and } x_2 \text{ are integer.}$$

The optimal table for the LP relaxation of this problem is given below.

	x_1	x_2	x_3	x_4	RHS
Z	0	0	-4/5	-18/5	88/5
x_1	1	0	-2/5	1/5	4/5
x_2	0	1	1/5	-3/5	8/5

a) **(3 pts)** Perform only <u>one</u> iteration of the cutting plane algorithm.

b) **(4 pts)** Solve the ILP by the branch & bound method.

2. **(5 pts)** Consider the following optimization problem.

Optimize $f(x) = x^3 - x^2 - x + 4$
s.t.
$x^2 \leq 9$
$x \geq -2$
$|x-1| \leq 3$

a) **(2.5 pts)** Find the stationary points and determine whether they are local minima or local maxima.

b) **(2.5 pts)** Find the global minima and global maxima.

3. **(4 pts)** Consider the following optimization problem.

Min $f(x_1,x_2) = 3x_1^2 + 2x_2^2$

s.t.

$x_1x_2 = 4$

a) **(1 pt)** Is the objective function convex, concave, both convex and concave, or neither convex nor concave? Why?

b) **(3 pts)** Solve the optimization problem.

4. **(4 pts)** Kuwait University buys size A4 papers for copy machines from a local supplier. The University uses 2,000 boxes of papers per week (1 week = 5 working days). The cost to order papers is 50 KD. The inventory holding cost is 0.2 KD per box of paper per week. Suppose that the University can get the following price discount:

No. of boxed ordered	Price per box
$q \leq 500$	6 KD
$500 < q \leq 1,500$	5 KD
$q \geq 1,500$	4 KD

a) **(3 pts)** Determine the optimal order size for papers.

b) **(1 pts)** If the lead time for ordering papers is 2 working days, what is the reorder point?

5. **(5 pts)** Consider the previous problem where Kuwait University buys size A4 papers with an ordering cost of 50 KD, an inventory holding cost of 0.2 KD per box of paper per week, and a lead time of 2 working days (1 week = 5 working days). Now, suppose that the weekly demand for papers is normally distributed with mean 2,000 boxes and standard deviation 500 boxes. The University desires to meet at least 95% of demand for papers from the stock. Determine the optimal order size and the reorder point for papers. ($P(Z>1.645) = 0.05$).

6. **(10 pts)** Suppose that an airline office has two phone operators answering calls for flight reservations. In addition, two callers can be put on hold until an operator becomes available to answer calls. If the four lines (the two operator lines and the two hold lines) are busy, then callers are lost. The time between arriving calls to the office has an exponential distribution with a mean of 6 minutes. The length of a caller's conversation for making reservation is exponentially distributed with a mean of 12 minutes.

a) **(1 pt)** If the first customer calls the office at 9:00 AM, what is the probability that the second customer will call between 9:06 and 9:12 AM?

b) **(1 pt)** What is the probability that a customer will call the office between 9:06 and 9:12 AM?

c) **(1 pt)** Given an operator began serving a customer 5 minutes ago, what is the probability that the customer's service will last another 10 minutes?

d) **(1 pt)** What is the probability that a caller will be put on hold?

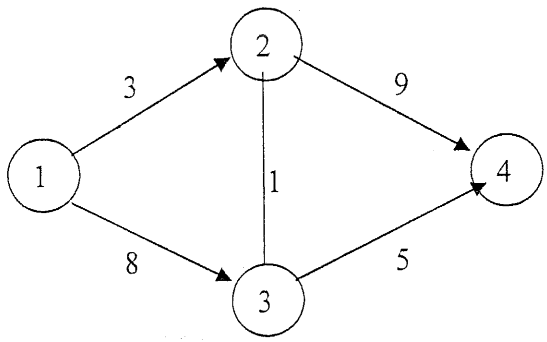
e) **(1 pt)** What is the expected number of busy operators?

f) **(1.5 pt)** Find the average number of calls on the hold lines.

g) **(1.5 pt)** Find the average time a caller spends on phone (talking to an operator or waiting on hold).

h) **(2 pt)** If each operator is paid 3 KD per hour and the value of each customer's time is 5 KD per hour, what is the expected hourly cost of the underlying queuing system?

7. **(5 pts)** Consider the following network where the number beside each arc shows the travel time.



Use backward recursion of dynamic programming to find the shortest route from node 1 to node 4 (Clearly define the stages, states, and the recursive relation).