Question 1: [10 Points]

The monthly energy bills for a school system over the past 10 years are available in ***Final_Exam_Q1.MTW.*** The energy bills are combined gas, oil and electric bills for the school system and are expressed in units 0f K.D. We would like to develop a time series regression model that could be used to obtain short-term forecasts of monthly energy bills.

a) Plot the time series.

b) From your plot, what kind of trend appears to exist?

c) Does seasonal variation appear to exist?

d) ***Define*** an appropriate time series regression model for this data.

e) Run a MINITAB program to fit the model you suggested. ***Write down*** the resulting model.

f) Forecast the monthly energy bills for the next three months. Construct 95% prediction intervals for the forecasts.

Question 2: [10 Points]

Consider the ARMA(1,1) process

$$x_t + 0.3x_{t-1} = \varepsilon_t - 0.4\varepsilon_{t-1} \qquad \{\varepsilon_t\} \text{ is Normal (mean} = 0, \sigma_t^2 = 1)$$

a) Verify whether it is stationary and/or invertible? Explain.

b) Find $\mu, \gamma_0, \rho_1, \rho_k \; k \geq 2$. Graph the theoretical autocorrelation function.

c) Simulate a series of 250 observations from the above model, then calculate and examine the sample autocorrelation function (ACF) for the simulated series. Write down the MINITAB commands used and the first 3 sample ACF obtained and compare them with the theoretical values obtained in part b. (use base = 5680)

Question 3: [10 Points]

The weekly demand for a small plastic container for the past 100 weeks is given in ***Final_Exam_Q3.MTW*** . The container is manufactured by an injection molding process and is widely used by several pharmaceutical houses as a package for a prescription drug.

a) Tentatively identify an appropriate model for this time series. Your analysis should include a logical explanation of the steps taken to find the chosen model.

b) ***Manually*** , find preliminary estimates of the model parameters.

c) ***Using Minitab***, estimate the parameters and write down the resulting model.

d) Forecast the demand for the next 5 weeks at origin 100.

Question 4: [10Points]

Consider the monthly energy bill data in ***Final_Exam_Q1.MTW.***

a) Tentatively identify an appropriate seasonal Box-Jenkins model for this time series. Your analysis should include a logical explanation of the steps taken to find the chosen model.

b) Estimate the parameters and write down the resulting model.

c) Compare the resulting seasonal Box-Jenkins model with the time series regression model in Question 1, based on the Residual Mean Square and the Akaike's Information Criterion (AIC).