

**Question 1:** [8 Points]

The quarterly energy bills for a school system over the past 10 years are available in *Final Exam\_Q1.MTW*. The energy bills are combined gas, oil and electric bills for the school system and are expressed in units of \$100. Since a plot of these data indicates the existence of a quadratic trend and constant seasonal variation, we would like to develop a time series regression model that could be used to obtain short-term forecasts of quarterly energy bills.

a) Define an appropriate time series regression model for this data.

b) Run a MINITAB program to fit the model you suggested. *Write down* the resulting model.

c) Perform diagnostic checks. Is the model adequate?

d) Forecast the quarterly energy bills for the next year.

**Question 2:** [12 Points]

Consider the ARMA(1,1) process

$$x_t + 0.3x_{t-1} = \varepsilon_t - 0.4\varepsilon_{t-1} \quad \{\varepsilon_t\} \text{ is Normal (mean} = 0, \sigma_t^2 = 1)$$

- a) Verify whether it is stationary and/or invertible? Explain.
- b) Find  $\mu, \gamma_0, \rho_1, \rho_k$   $k \geq 2$ . Graph the theoretical autocorrelation function.

c) Express the model in an MA representation if it exists.

d) Express the model in an AR representation if it exists.

- e) Simulate a series of 250 observations from the above model, then calculate and examine the sample autocorrelation function (ACF) for the simulated series. Write down the MINITAB commands used and the first 3 sample ACF obtained and compare them with the theoretical values obtained in part b. (use base = 5680 )

**Question 3:** [10 Points]

The weekly demand for a small plastic container for the past 100 weeks is given in *Final\_Exam\_Q3.MTW*. The container is manufactured by an injection molding process and is widely used by several pharmaceutical houses as a package for a prescription drug.

- a) Tentatively identify an appropriate model for this time series. Your analysis should include a logical explanation of the steps taken to find the chosen model.
- b) *Manually*, find preliminary estimates of the model parameters.
- c) *Using Minitab*, estimate the parameters and write down the resulting model.
- d) Forecast the demand for the next 5 weeks at origin 100.







**Question 4:** [10Points]

Consider the quarterly energy bill data in *Final\_Exam\_Q1.MTW*.

- a) Tentatively identify an appropriate seasonal Box-Jenkins model for this time series. Your analysis should include a logical explanation of the steps taken to find the chosen model.
- b) Estimate the parameters and write down the resulting model.
- c) Compare the resulting seasonal Box-Jenkins model with the time series regression model in Question 1, based on the Residual Mean Square and the Akaike's Information Criterion (AIC).



