Question 1 [5 points]

Let $X_1, X_2, .., X_n$ be a random sample from uniform$(0,\theta)$, where θ is unknown. Let $Y_n = \max\{X_1, X_2, .., X_n\}$. Show that the estimate Y_n converges in probability to θ, (i.e., $Y_n \xrightarrow{p} \theta$.)

<u>Question 2</u> [6 points]

Let $X=(X_1,X_2,X_3,X_4)^t$ have multivariate normal distribution with mean vector 0 and variance-covariance metrix

$$\Sigma=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Let $Y=\frac{1}{4}\sum_{i=1}^{4}X_i$. What is the distribution of Y.

Question 3 [8 points]

Let W denote a random variable that is $\chi^2(2)$. Let V denote a random variable that is also $\chi^2(2)$, and let W and V be independent.

1) What is the joint pdf of W and V.

2) Define a new random variable by letting $Y=\frac{W}{V}$, using the change-of-variable technique, find the pdf of the random variable Y.

Question 4 **[8 points]**

Let $Y_1<Y_2<Y_3$ be the order statistics of a random sample of size 3 from the distribution having pdf $f(x)=1$, where $0<x<1$, zero elsewhere.

1) Find $P(Y_1>\frac{1}{4})$.

2) Find the joint pdf of Y_2 and Y_3.

3) Is $Z_1=Y_2$ and $Z_2=Y_3-Y_2$ independent (Show your work).

Question 5 [12 points]

Let X and Y have the joint pdf

$$f(x,y) = \begin{cases} cy & ,0 < x < y < 1 \\ 0 & ,otherwise. \end{cases}$$

1) Find the constant c.

2) Find the correlation coefficient of X and Y.

3) Are X and Y independent?

4) Find $f_{y|x}(y|x)$.

5) Find the distribution of $V=\mathrm{E}[Y|X]$.

6) Find $\mathrm{E}[V]$ and $\mathrm{Var}[V]$.