Stat 310, Midterm I, April 8, 2009

I. A. Suppose N has a Poisson distribution with parameter μ. Assume conditional on N, a random variable M has a binomial distribution with parameters N and p. Show that the marginal distribution for M is a Poisson Distribution with mean μp. (4 points)

B. Assume random variables N and M are as in Part A. Find $P\{N = n, \quad M = 0\}$. (2 Points).

II. A Markov chain has transition probability matrix

$$\mathbb{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 1/5 & 2/5 & 2/5 \\ 1/4 & 1/4 & 2/4 \\ 1/7 & 2/7 & 4/7 \end{array} \right\| \end{array}$$

Assume the initial distribution is $p_0 = p_1 = 2/9$, $p_2 = 5/9$.

A. Find $P_{21}^{(3)}$. (2 Points).

B. Find $P\{X_3 = 1, X_1 = 0 \mid X_0 = 2\}$. (2 points).

C. Find $P\{X_3 = 1 | X_0 = 2, X_2 = 3\}$. (2 points).

III. A Markov chain has transition probability matrix

$$\mathbb{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \left\| \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 1/4 & 1/4 \\ 1/5 & 2/5 & 0 & 2/5 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right\|$$

A. Find the absorbing probabilities to the absorbing states, given $X(0) = 3$. Verify that sum of the absorbing probabilities equal 1. (4 Points)

B. Find the expected time absorbtion at state 0, given $X(0) = 3$. (3 Points).

IV. A Markov chain has transition probability matrix

$$\mathbb{P} = \begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \left\| \begin{array}{ccccc} 1/4 & 1/4 & 1/4 & 1/4 & 0 \\ 1/3 & 0 & 0 & 0 & 2/3 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/5 & 2/5 & 2/5 \\ 0 & 0 & 3/7 & 4/7 & 0 \end{array} \right\| \end{array}$$

A. Specify the two Communicative classes. (2points)

B. Which communicative class is recurrent, and which one is transient? (2 points).

C. Find the stationary distribution for the recurrent class. (3 points).

V.

A. Consider a Success Run model with infinite states in which

$$p_i = 1 - (\frac{1}{2})^{i+1}, \quad i = 0, 1, 2, \cdots$$

Success Run

Determine if the ~~random walk~~ is positive recurrent, or null recurrent. Find the stationary distribution of the ~~random walk~~. (4 points)

Success Run

B. A Markov chain has transition probability matrix

$$\mathbb{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left\| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 3/4 & 0 & 1/4 & 0 \\ 0 & 1/5 & 0 & 4/5 \\ 1/6 & 5/6 & 0 & 0 \end{array} \right\| \end{array}$$

Specify $f_{33}^{(n)}$ for $n = 1, 2, 3, \ldots$ Is the state 3 recurrent or transient? provide reasons for your answer. (3 points)

VI. Let $\{X(t),\ t \geq 0\}$ be a Poisson process with intensity parameter λ. Find

A. $P\{X(t) = j | X(s) = i\},\ j \geq i; s < t$. (2 Points)

B. $E[X(s)X(t)],\quad s < t$. (3 Points)

C. $P\{X(r) = i, X(s) = j, X(t) = k\},\ \ 1 \leq j \leq k;\ r < s < t$. (2 Points)