1. **(6 pts)** Consider the following maximizing LP problem whose TORA output is given below.

Max $Z = 300x_1 + 400x_2 - 50x_3$

Subject to:

$$
\begin{aligned}
0.8x_1 + x_2 - x_3 &\leq 0 \\
x_3 &\leq 98 \\
0.6x_1 + 0.7x_2 &\leq 73 \\
2x_1 + 3x_2 &\leq 260 \\
x_1 &\geq 88 \\
x_2 &\leq 26
\end{aligned}
$$

and $x_1, x_2, x_3 \geq 0$

a) **(1 pt)** What are the optimal values for decision variables x_1, x_2, x_3 and the objective function Z?

b) **(1 pt)** What is the shadow price for the RHS of each constraint?

c) **(1.5 pts)** If the objective function coefficient of x_1 changes from 300 to 100, what would be the new optimal values for x_1, x_2, x_3 and Z?

d) **(1.5 pt)** If the RHS of the second constraint changes from 98 to 96, what would be the new optimal value for Z?

e) **(1 pt)** What is the range of feasibility for the RHS of the last constraint? What does this range show?

2. **(5 pts)** Consider the following minimizing transportation problem. The supplies at sources 1, 2 and 3 are respectively 10, 10 and 15, and the demands at destinations 1, 2 and 3 are 10, 5 and 15. Any excess supply at sources must be stored with storage costs of 4, 2, and 3 per unit at sources 1, 2, and 3, respectively. No shipment is allowed from source 2 to destination 3.

	Destination		
Origin	1	2	3
1	9	6	7
2	5	5	-
3	8	10	9

a) **(2.5 pts)** Formulate this problem as an LP to minimize the total transportation and storage cost.

b) **(2.5 pts)** Use the transportation simplex method to solve the problem.

3. **(4 pts)** A department at Kuwait University wants to know how to assign four professors A, B, C, and D to four courses 1, 2, 3, and 4. Professor A can be assigned to at most two courses while each of the rest of professors can be assigned to at most one course. Each course needs to be assigned to a professor. On a scale of 1 (low) to 10 (high), the interest of a professor in teaching a course is given in the table below. The objective is to determine which professor should be assigned to which course to maximize the professors' total teaching interest. Use the Hungarian method to solve this assignment problem.

	Course			
Professor	1	2	3	4
A	8	3	6	9
B	10	6	7	5
C	4	7	5	10
D	9	8	2	9

4. **(4 pts)** Formulate the following problem as an integer linear programming model.

Min $Z = x_1 + 3f(x_2)$

where

$$f(x_2) = \begin{cases} 5 + 4x_2, & \text{if } x_2 > 0 \\ 0, & \text{otherwise} \end{cases}$$

subject to:

(i) 2 of the following 3 constraints hold.

$x_1 + x_2 \leq 10$

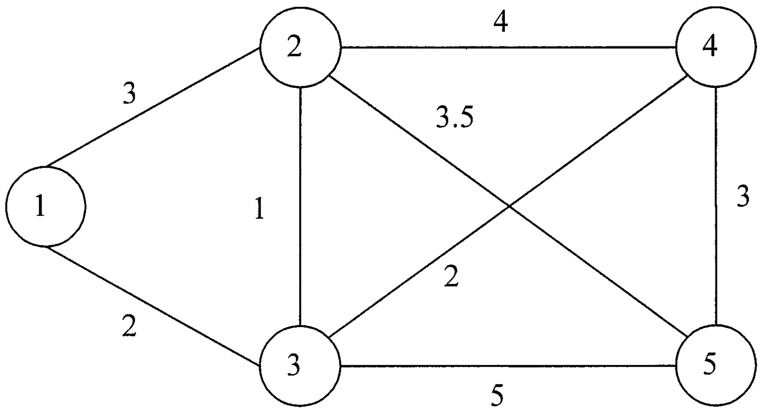
$x_1 - x_2 \leq 4$

$x_1 + 2x_2 \leq 8$

(ii) $x_1 = -1$, or 1, or 2.

(iii) Either $x_1 \leq 6$ or $x_2 \leq 4$.

5. **(5 pts)** Consider the following network of five computers where the number beside each arc shows the length of cable required to connect the respective computers.



a) **(2.5 pts)** How the five computers can be connected directly/indirectly to minimize the total length of cable needed?

b) **(2.5 pts)** Use Dijkstra algorithm to find the minimum length of cable needed to connect only computer 1 to computer 5.

6. **(5 pts)** Consider a project whose activities, their immediate predecessors, and the three estimates for the duration of each activity are given in the following table.

Activity	Immediate predecessors	Optimistic time	Most likely time	Pessimistic time
A	--	3	5	7
B	--	4	5	6
C	A,B	1	5	9
D	B	6	6	6
E	C,D	5	8	11

a) **(1.5 pts)** Draw the project network.

b) **(1 pt)** Compute the mean and variance for the duration of each activity.

c) **(2.5 pts)** Write the probability statement for computing the probability of completing the project by the deadline of 20.

7. **(6 pts)** Consider the following two-person-zero-sum game where the payoff table is given for player I.

		II		
	Strategy	1	2	3
I	1	5	1	0
	2	0	4	4

a) **(1 pt)** Identify and eliminate the dominated strategies for each player as far as possible.

b) **(1.5 pts)** Apply the minimax/maximin criterion to find the pure strategy (if any) for each player. Does the game have a stable solution? Why?

c) **(1.5 pt)** Using the reduced payoff table of part (a), write a LP problem to find the mixed strategies for player II.

d) **(1.5 pt)** Solve the LP of part (c) to compute the optimal mixed strategies for player II.

8. **(5 pts)** Consider the following network where the number beside each arc shows the travel time. Use backward recursion of dynamic programming to find the shortest route (including its minimum travel time) from node 1 to node 6. Clearly define the stages, states, and the recursive relation.