1. **(5 pts)** Consider the following LP problem.

Min $Z = x_1 - x_2$

s.t.

$x_1 + x_2 \geq 0$

$x_1 - x_2 \leq 1$

$-x_1 + x_2 = 1$

$x_2 \geq -1$

x_1 is free.

a) **(3 pts)** Solve the problem graphically and comment on its solution.

b) **(2 pts)** Compute the shadow price for the RHS of the second constraint. Explain the meaning of this shadow price.

2. (6 pts) Consider the problem of loading six items into a knapsack. The weights, volumes, and values of the items are given below.

Item	Unit weight (kg)	Unit volume (ft^3)	Unit worth (KD)
1	6	0.3	3
2	10	0.5	6
3	3	0.7	2
4	8	0.9	9
5	5	0.2	5
6	7	0.4	8

A combination of 3 to 5 items must be loaded into the knapsack. The maximum allowable weight and volume of the knapsack are 25 kgs and 3 ft^3, respectively. In addition,

(i) If item 1 is loaded, item 4 must also be loaded.

(ii) If item 2 is loaded, item 5 must not be loaded.

(iii) Items 1 or 3 or 6 must be loaded.

Formulate this as a binary ILP to maximize the total value of the knapsack's contents.

3. **(5 pts)** The LP formulation of a problem and its TORA output are given below.

$$\text{Min } Z = 2x_1 + x_2 + 3x_3$$

$$\begin{aligned} \text{s.t.} \quad & x_1 + 2x_2 \geq 4 \\ & x_2 - x_3 \leq 2 \\ & x_1 + 2x_2 + 3x_3 = 6 \\ \text{and} \quad & x_1, x_2, x_3 \geq 0 \end{aligned}$$

where x_1, x_2, and x_3 show the numbers of products 1, 2, and 3. The objective function shows cost in KD.

a) **(1 pt)** How many units of products 1, 2, and 3 should be produced to minimize cost?

b) **(1 pt)** What is the shadow price for the RHS of each constraint?

c) **(1 pt)** If the unit cost of product 1 increases from 2 KD to 8 KD, how many products 1, 2, and 3 will be produced and what would be the minimum cost?

d) **(1 pt)** If the RHS of constraint 2 decreases to 1.5, what would be the minimum cost?

e) **(1 pt)** Give the range of optimality for the unit cost of product 1. What does this range indicate?

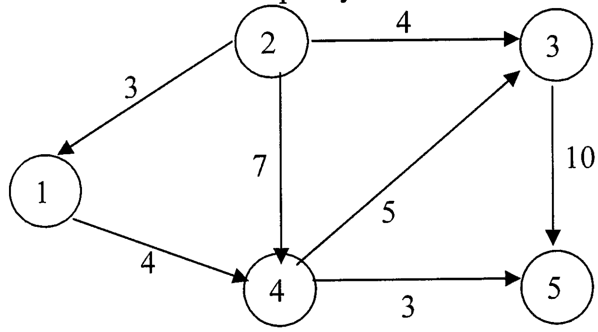
4. **(6 pts)** Consider the following minimizing transportation problem in which no items can be shipped from source A to destination 2. The inventory cost per unit at sources A and B are 2 KD and 3 KD, respectively.

	Destination		
Source	1	2	Supply
A	8	-	10
B	4	5	20
Demand	10	15	

a) **(3 pts)** Generate an initial solution by using the north west corner rule.

b) **(3 pts)** Use the initial solution of part (a) and the transportation simplex method to find the optimal solution.

5. **(3 pts)** Use the maximum flow algorithm to find the maximum flow from node 2 to node 5. The number beside each arc shows the arc capacity.



6. **(6 pts)** Consider a project whose activities and their immediate predecessors as well as activity durations are given in the following table.

Activity	Immediate Predecessors	Duration (days)
A	--	3
B	--	5
C	A,B	6
D	A,B	4
E	C,D	5
F	C	2

a) **(2 pts)** Draw the project network.

b) **(2 pts)** Find the critical path.

c) **(2 pts)** How much activity A can be delayed without delaying the project completion time? How about activity F?

7. **(5 pts)** Consider the following two-person-zero-sum game where the payoff table is given for player I.

	Strategy	II: 1	2	3
	1	5	0	-2
I	2	1	4	2
	3	0	4	0

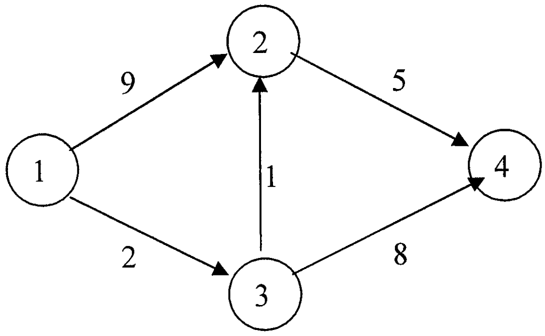
a) **(1 pt)** Identify and eliminate the dominated strategies for each player as far as possible.

b) **(1 pt)** Apply the minimax/maximin criterion to the reduced payoff table of part (a) to find the best strategy (if any) for each player. Does the game have a stable solution? Why?

c) **(2 pts)** Using the reduced payoff table of part (a), write a LP problem to find the mixed strategies for player I.

d) **(1 pt)** Solve the LP of part (c) to compute the optimal mixed strategies for player I.

8. **(4 pts)** Consider the following network where the number beside each arc shows the distance. Use backward recursion of dynamic programming to find the shortest route (including its minimum distance) from node 1 to node 4. Make sure that you clearly define the stages, states, and the recursive relation.



5. **(4 pts)** Three persons A, B, and C are to be assigned to four jobs 1, 2, 3, and 4. Person A can be assigned to any two jobs while each person B and C can be assigned to any one job. Each job must be assigned to a person. The following table provides the training cost for each person for a job..

	Job			
Person	1	2	3	4
A	6	9	8	3
B	5	7	6	10
C	10	5	7	4

Apply the Hungarian method to determine which person should be assigned to which job to minimize the total cost of training.