

Question 1. (7 marks)

If S_1^2 and S_2^2 are the variances of independent random samples of size $n_1=25$ and $n_2=41$ are taken from two normal populations having the means $\mu_1=5$ and $\mu_2=10$ and variances $\sigma_1^2=49$ and $\sigma_2^2=36$. Let \bar{x}_1 be the average of the random sample of size $n_1=25$. Find

a) $P(3 < \bar{x}_1 < 7)$ (3 marks)

b) $P(S_1^2 > 25 \cdot 32)$ (2 marks)

c) $P(\frac{S_1^2}{S_2^2} < 3 \cdot 12)$ (2 marks)

Question 2. (6 marks)

Let X_1, X_2, \dots, X_n be a random sample from the distribution with density

$$f(x, \theta) = (\theta + 1) x^\theta, 0 < x < 1$$

where $\theta > -1$. A random sample of 5 yielded data :

$$x_1 = 0.92, x_2 = 0.79, x_3 = 0.90, x_4 = 0.65 \text{ and } x_5 = 0.86 .$$

a) Find an estimator for θ by using the method of moment, then compute the estimate for these data. (3 marks)

b) Find an estimator for θ by using the maximum likelihood method. (3 marks)

Question 3. (8 marks)

Part I: (4 marks)

A study of two kinds of photocopying equipment shows that 61 failures of the equipment A took on the average 80.7 minutes to repair with a standard deviation of 19.4 minutes, whereas 61 failures of equipment B took average 88.1 minutes to repair with a standard deviation of 18.8 minutes.

a) Construct a 98% confidence interval for the ratio of the variances of the two populations sampled.(2 marks)

b) Test the hypothesis that the population standard deviation of equipment A is equal to 20 against the alternative it is less than 20 at significance level of 0.05(2 marks)

Part II(4 marks)

A single observation of a random variable, X, having the exponential distribution

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0$$
 is used to test

$$H_0 : \theta = 2 \quad \text{against} \quad H_1 : \theta = 7$$

If the null hypothesis is rejected iff the value of the random variable is greater than 4.

1) Find the probability of rejecting H_0 when it is true .(2 marks)

2) Find the power of the test (2 marks)

Question 4 (5 marks)

In random samples of 250 persons with low income, 200 persons with average incomes, and 150 persons with high incomes, there were, respectively, 155, 118, and 87 who favor a certain piece of legislation. Use the 0.05 level of significance to test whether the responses is affected by the income or not.

Question 5 (7 marks)

The following are the scores that 10 students obtained on the midterm and final examinations in a course in statistics:

Midterm examination X	71	49	80	73	93	58	82	64	32	85
Final examination Y	83	62	76	77	89	48	78	76	51	74

Where $\sum x = 687$, $\sum x^2 = 50253$, $\sum y = 714$, $\sum y^2 = 52600$, $\sum xy = 50875$

- a) Find the equation of the least squares line that will enable us to predict a student's final examination score in the course on the basis of his or her score on the midterm examination.(3 marks)

- b) Predict the final examination score of the student who received an 84 on the midterm examination.(1 mark)

- c) Calculate the correlation coefficient **then** comment. (3 marks)

Question 6 (7 marks)

The following are the numbers of words per minute that a secretary typed on several occasions on four different typewriters:

	Typewriter1	Typewriter 2	Typewriter 3	Typewriter 4
	71	68	75	60
	75	66	81	63
	69	69	78	59
	77	62	73	62
mean	73	66.25	76.75	61

Where $\bar{X} = 69.25$, $S^2 = 47$

a) Complete the following ANOVA table. (5 marks)

<i>Source of Variation</i>	<i>Degrees of Freedom</i>	<i>Sum of Square</i>	<i>Mean Square</i>	<i>f</i>
Treatments		589.5		
Errors		115.0		
Total	15			

b)Test whether the differences among the four sample means can be attributed to chance. Use the 0.05 level of significant. (2 marks)