

1. [5 Marks] A large lot of electronic units contains 10% defective units. Assume that we randomly select units from the lot one by one. Find

a) The probability that 6 units must be selected to get the first defective unit.

b) The probability that 10 units must be selected to get the 4<sup>th</sup> defective unit.

2. [5 Marks] Assume  $E(X_1) = 3$ ,  $E(X_2) = 4$ ,  $V(X_1) = 2$ ,  $V(X_2) = 5$  and  $C(X_1, X_2) = 3$ . Find

a)  $E(2X_1 - 5X_2 + 14) =$

b)  $V(5X_1 - 3X_2 + 7) =$

3. [5 Marks] Assume that  $P(A) = 0.5$ ,  $P(B) = 0.3$ .

a)  $P(A \cup B) + P(A \cap B) =$

b) If  $A$  and  $B$  are independent, then

$$P(A \cup B) =$$

4. [7 Marks]  $X$  and  $Y$  are continuous random variables with

$$f(x, y) = \begin{cases} \frac{1}{2}x + \frac{3}{2}y & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find

a)  $P\{X \leq 0.8 \text{ and } Y \leq 0.6\} =$

b) The marginal probability density  $g(x)$  of  $X$ .

c) The conditional probability density of  $Y$  given  $X = x$  (i.e.,  $w(y|X = x)$ ).

5. [4 Marks] Assume  $X$  has the Geometric distribution

$$g(x) = (1 - \theta)^{x-1} \theta, x = 1, 2, \dots$$

with parameter  $\theta$ . Prove that

$$M_X(t) = \frac{\theta e^t}{1 - (1 - \theta) e^t}$$

[Hint:  $1 + \lambda + \lambda^2 + \dots = \frac{1}{1-\lambda}$ , for  $|\lambda| < 1$ ].

6. [5 Marks] Assume  $X$  is a continuous  $RV$  with density function

$$f(x) = \begin{cases} \frac{1}{4}x & \text{for } 0 < x < 2 \\ \frac{1}{4}(4-x) & \text{for } 2 \leq x < 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Find

a)  $F(1) =$

b)  $F(3) =$

c)  $P(1 < X \leq 3) =$

7. [3 Marks] Assume  $X$  has the Normal distribution with mean 3 and variance  $\sigma^2$ . Find the value of  $\sigma$  such that

$$P(X > 5.3) = 0.125.$$

8. [4 Marks] The distribution function of the discrete random variable  $X$  is

$$F(x) = \begin{cases} 0 & , x < -3 \\ 0.1 & , -3 \leq x < 0 \\ 0.4 & , 0 \leq x < 3 \\ 0.75 & , 3 \leq x < 5 \\ 0.9 & , 5 \leq x < 6 \\ 1 & , x \geq 6. \end{cases}$$

Find

a)  $P(X = 0) =$

b)  $P(0 \leq X \leq 5) =$

9. [5 Marks] Bowl A contains 3 red chips and 7 blue chips. Bowl B contains 6 red chips and 4 blue chips. One Bowl is randomly selected and then one chip is randomly selected from this Bowl.

a) What is the probability that this chip is red?

b) Given that the selected chip is red, what is the probability that it is drawn from Bowl B?

10. [7 Marks] The j.p.d. of the discrete random variables  $X$  and  $Y$  is

	$y$	-1	0	1	
$x$					
0		$\frac{1}{3}$	$\frac{1}{9}$	0	
1		$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	

Find

a) The marginal probability density of  $X$

$x$	0	1

b) The conditional p.d. of  $Y$  given  $X = 1$

$y$	-1	0	1
$w(y 1)$			

c)  $E\{Y|X = 1\} =$

d)  $C(X, Y) =$