

Q1 (9 marks) . The joint distribution of (X, Y) is defined by:

	0	1	2
1	0.1	0.1	0
2	0.1	0	0.3
3	0.2	0.1	0.1

(a) Find the marginal probability functions of X and Y .

(b) Find the covariance of X and Y .

(c) Find $\text{var}(X - 2Y)$.

Q 2 (8 marks) . Consider the joint probability density function

$$f(x, y) = 4xy \quad 0 < x < 1, 0 < y < 1.$$

Find:

(a) The joint distribution $F(1/2, 1/4)$,

(b) $p(X \geq 1/2, Y \geq 1/4)$.

(c) The conditional probability function of Y given that $X = x$, $w(y|X = x)$.

(d) $E(Y|X = \frac{1}{2})$

Q 3 (8 marks). Consider the moment generating function $M(t) = \exp(2t + t^2)$ of a random variable X and answer the following questions:

(a) What is the probability distribution of X ?

(b) Find the moment generating function of $Y = 3X + 2$.

(c) What is the probability distribution of Y ?

(d) Find the mean and the variance of Y .

Hint: For $N(\mu, \sigma)$, $M(t) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2)$.

Q 4 (3 marks) . (a) A random experiment has two possible outcomes { success, failure} such that the probability of success is 0.25. This experiment is repeated 40 times. Assume that these trials are independent. Let X denote the number of successes, find $p(X \geq 5)$.

Q5. (8 marks)A large lot of electronic units contains 10 % of defective units. Let X denote the number of units selected to obtain first defective units. Find

(a) $p(X > n)$

(b) Probability that 5 units must be selected to get 1 defective unit.

(c) Consider a geometric distribution with parameter θ and prove that the moment generating function equals

$$M_X(t) = \frac{\theta e^t}{1 - e^t(1 - \theta)}$$

Hint: $\sum_{n=1}^{\infty} x^n = \frac{x}{1-x}$ for $|x| < 1$.

Q6. (2 marks) For $x > 0$, consider the cdf:

$$F(x) = 1 - (1 + x)e^{-x}$$

and find

$$p(X > 0.5).$$

Q7. (4 marks) The probability density function of a random variable X is given by

$$f(x) = \frac{c}{\sqrt{x}} \quad \text{for } 0 < x < 4.$$

(a) Find the value of c ;

(b) Define the events $A = (1 < X < 3)$ and $B = (2 < X < 4)$ then find $p(A \cup B)$.

Q8. (4 marks). The number of individuals arrive to a bus station within an hour is a *r.v.* that follows Poisson distribution with an average equals 5 individuals per hour. Find the probability that

(a) At least 3 passengers will arrive in the next hour.

(b) No passengers will arrive in the next 2 hours.

Q9 (4 marks). Bowl I contains 3 red chips and 7 blue chips. Bowl II contains 6 red chips and 4 blue chips. A bowl is selected at random and then 1 chip is drawn from this bowl.

(a) find the probability that the chip is red.

(b) Relative to the hypothesis that the chip is red, find the probability that it is drawn from bowl II.